Introduction to the spatial equilibrium model

Melanie Morten, Stanford

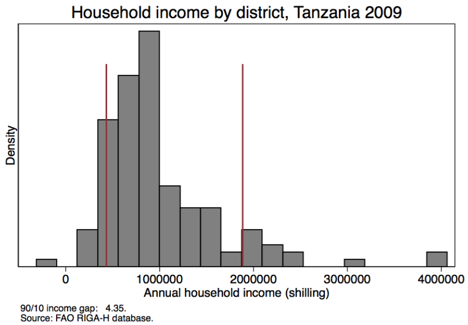
April 22, 2021

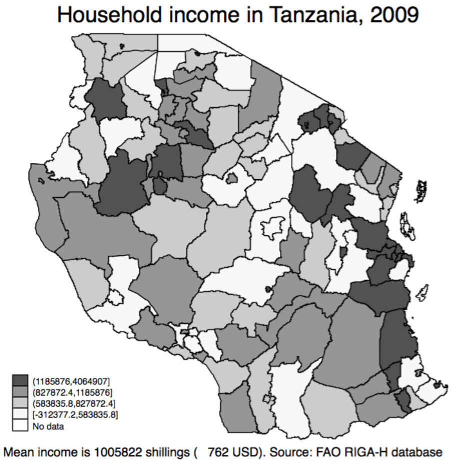
What explains the coexistence of this within same country?

(a) Jakarta

(b) Rural Java

Income distribution across space (Tanzania)





Distribution of wages in the US

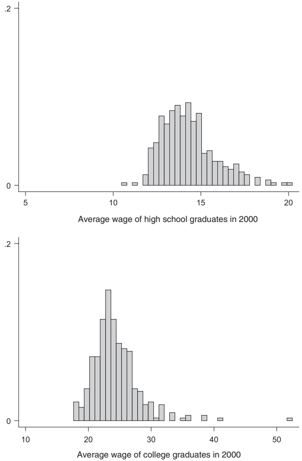


Figure 2 ***Distribution of average hourly nominal wage of high school graduates and college graduates, by metropolitan area.*** Notes: This figure reports the distribution of average hourly nominal wage of high school graduates and for college graduates across metropolitan areas in the 2000 Census of Population. There are 288 metropolitan areas. The sample includes all full-time US born workers between the age of 25 and 60 who worked at least 48 weeks in the previous year.

Moretti (2011)

Today's class: introduction to basic spatial equilibrium model

- Types of research questions
  - What explains the difference in income across looations?
  - Welfare/distribution of welfare when a location gets a productivity shock?
  - Welfare/distribution of welfare when roads are built?
- Workhorse model: Rosen-Roback
  - We'll work through $n = 2$ case to develop intuition
- Model extensions
  - $n > 2$ regions
  - Other applications (trade, commuting, etc.)
- How to solve on computer
  - Sally Zhang (PhD student, Stanford) will go through key equations + show simulations in TA section

Why aren't wages equalized across space?

- People maximize utility, not wages
- Places may have different levels of amenities
- Places may have different costs of living
- Places may have differently skilled workers (selection)
- People also usually differ over their individual preferences for locations
- Additionally, may have frictions
    - Migration costs
    - Trade costs
    - Housing

Outline

Rosen-Roback model: Exogenous prices

- 2 locations
- N locations

RR model: Endogenous prices

Extend to different settings

Empirical exercise

Wrap up

Outline

Rosen-Roback model: Exogenous prices

- 2 locations
- N locations

RR model: Endogenous prices

Extend to different settings

Empirical exercise

Wrap up

Simple example: 2 locations

- Assume wages, rents, amenities are exogenous
- Person i's indirect utility of being in A:

$$V_A^i = \underbrace{\mathbf{w}age_A - \mathbf{r}ent_A + \mathbf{A}menities_A}_{\text{common to A } (V_A)} + \epsilon_A^i$$

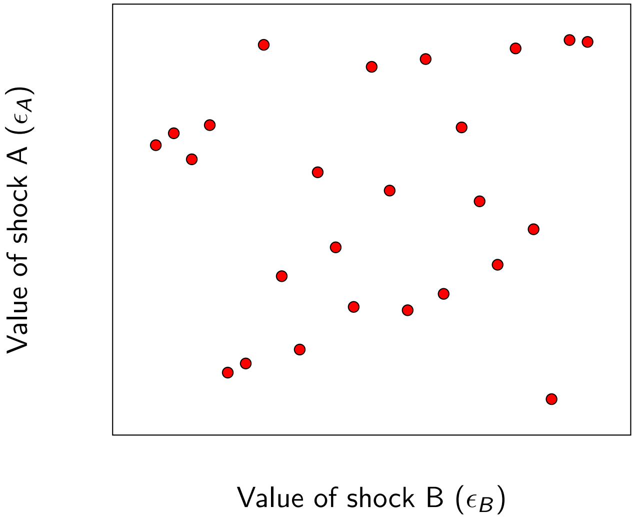
- Person i's indirect utility of being in B:

$$V_B^i = \underbrace{\mathbf{w}age_B - \mathbf{r}ent_B + \mathbf{A}menities_B}_{\text{common to B } (V_B)} + \epsilon_B^i$$

Check intuition: if people can freely move, what is equalized across space?

- Wages?
- Rents?
- Observed utility (i.e., V_A, V_B)?
- Average welfare (i.e., $V_A + \epsilon_A$, $V_B + \epsilon_B$)?

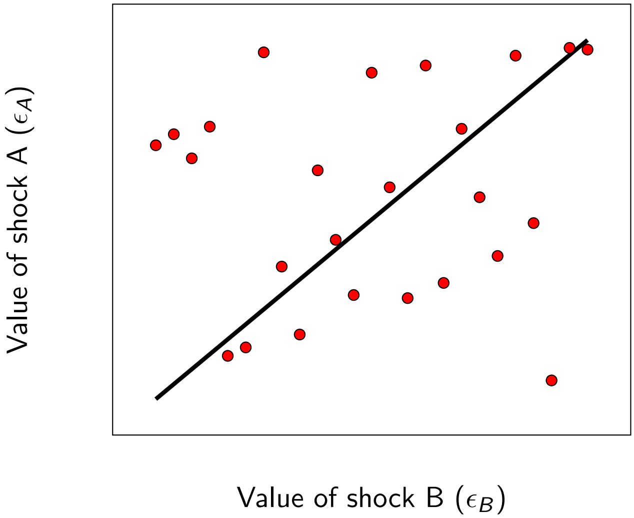
Migration decision: choose location that maximizes utility



Live in A if:

$$V_A + \epsilon_A > V_B + \epsilon_B$$
$$\epsilon_A > \epsilon_B + (V_B - V_A)$$

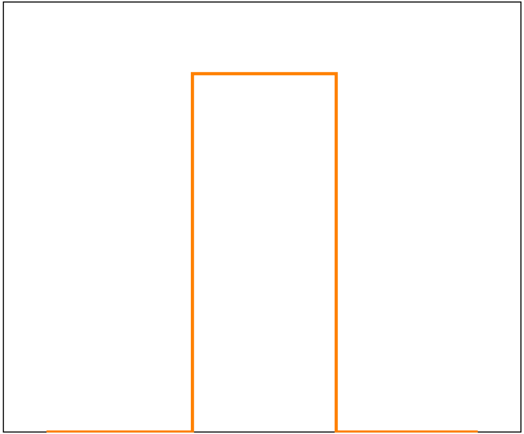
Migration decision: choose location that maximizes utility



Live in A if:

$$\begin{aligned} V_A + \epsilon_A &> V_B + \epsilon_B \\ \epsilon_A &> \epsilon_B + (V_B - V_A) \end{aligned}$$

Migration choice - uniform distribution assumption



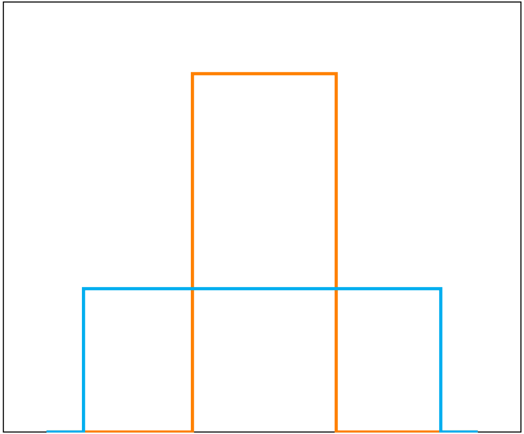
$\epsilon_A - \epsilon_B$

- Assume that the difference in shocks is uniform $\epsilon_B - \epsilon_A \sim U(-S, S)$:
- Live in A if:

$$V_A + \epsilon_A > V_B + \epsilon_B$$
$$V_A - V_B > \epsilon_B - \epsilon_A$$

- Note the role of S here
  - People are less responsive to differences in indirect utility
  - Changes labor supply elasticity
  - Sometimes referred to as a migration cost

Migration choice - uniform distribution assumption



$\epsilon_A - \epsilon_B$

- Assume that the difference in shocks is uniform $\epsilon_B - \epsilon_A \sim U(-S, S)$:
- Live in A if:

$$V_A + \epsilon_A > V_B + \epsilon_B$$
$$V_A - V_B > \epsilon_B - \epsilon_A$$

- Note the role of S here
  - People are less responsive to differences in indirect utility
  - Changes labor supply elasticity
  - Sometimes referred to as a migration cost

Spatial equilibrium: what share of people live in each location?

- Person i will choose to live in A if:

$$V_A + \epsilon_A^i > V_B + \epsilon_B^i$$
$$\rightarrow \epsilon_B - \epsilon_A < V_A - V_B$$

- Overall share of the population who live in A:

$$\begin{aligned} P(\epsilon_B - \epsilon_A < V_A - V_B) &= F_{\epsilon_B - \epsilon_A}(V_A - V_B) \\ &= \frac{V_A - V_B + s}{2s} \\ &= \frac{1}{2} + \frac{V_A - V_B}{2s} \end{aligned}$$

Uniform distribution on [a,b]: $F(x) = \frac{x-a}{b-a}$

Is this a spatial equilibrium?

- If $V_A > V_B$, more people will live in A than B
- Despite the different Vs, we have a spatial equilibrium
  - The *marginal* migrant (not the *average* one) is indifferent between A and B
  - i.e., no-one wants to live elsewhere
- Note: usually observe equivalent of V_A, V_B, but that's not welfare
  - Need to account for the idiosyncratic shock i.e., $E(V_A + \epsilon_A|\text{choose A})$ vs $E(V_B + \epsilon_B|\text{choose B})$
  - In simple case: hard to check since defined the difference in shock
  - But often do get this property (e.g., with Frechet) – in which case, average welfare is equalized across the two locations, despite the difference in V_A, V_B.

Outline

Rosen-Roback model: Exogenous prices

- 2 locations
- N locations

RR model: Endogenous prices

Extend to different settings

Empirical exercise

Wrap up

How to extend to more than 2 locations?

- Model generalizes easily by assuming extreme value shocks
  1. Gumbel (Type 1): used in many IO and urban models - additively separable; often used for preference shocks

$$F(x) = e^{-e^{-x+\alpha}}$$

$$\max_i v_i + \epsilon_i \quad \epsilon_i \sim EV1.$$

  2. Frechet (Type 2): used in many trade and the migration models; often used for productivity shocks Eaton and Kortum (2002)

$$F(x) = e^{-x^{-\theta}}$$

$$\max_i v_i \epsilon_i, \quad \epsilon_i \sim EV2.$$

Extreme value magic

- Extreme value shocks are commonly used because have closed form solutions

$$\text{Gumbel}: p(\text{choose i}) = \frac{e^{v_i}}{\sum_i e^{v_i}}$$

$$\text{Frechet}: p(\text{choose i}) = \frac{v_i^{\theta}}{\sum_i v_i^{\theta}}$$

- Can derive these expressions, also expected values conditional on choosing i - get nice closed form solutions so very convenient to work with
- General intuition of simple model goes through

Outline

Rosen-Roback model (Rosen, 1979; Roback, 1982)

Baseline assumptions:

- Each city produces homogenous good that is freely traded (i.e., consumption cost the same across locations)
- Fixed amount of land
- Labor perfectly mobile
- Capital perfectly mobile (or, no capital in production fn. These are equivalent, why?)

Note: initial RR model did not have individual heterogeneity. Added by Bayer et al. (2007). See Moretti (2011) for a textbook treatment.

RR model

Migration driven by 4 factors:

1. Wages
2. Cost of living (rents)
3. Amenities
4. Individual preferences

Definition of a spatial equilibrium

Given the economic environment (exogenous productivities and amenities), a spatial equilibrium (number of workers, housing rent, wage) solves the following equations:

- Labor supply: individuals choose location to maximize utility

$$\max_d w_d - R_d + A_d + \epsilon_d^i$$

- Labor demand: firms pay workers their marginal product (here: assume only labor. Equivalent to assuming CRS in capital, labor with international price of capital)

$$Y_d = X_d N_d$$
$$w_d = X_d$$

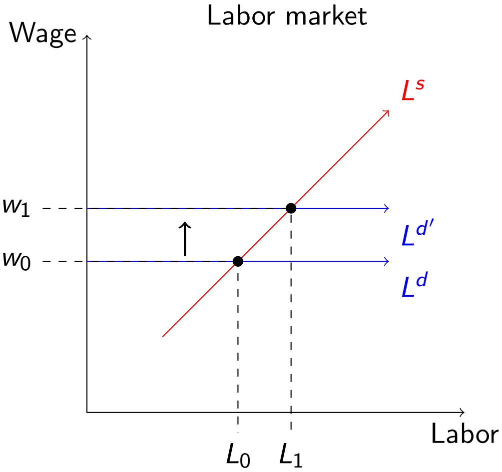
- Housing demand: each worker demands one unit of housing
- Housing supply: allow housing elasticity k_b

$$r_b = z + k_b N_b$$

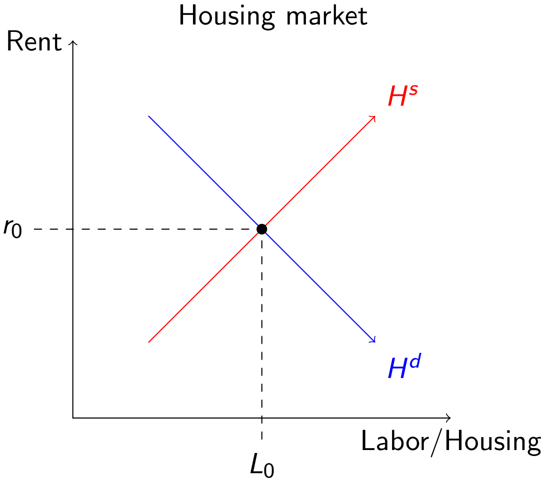
Consider a productivity increase in location A

- Wages increase in A
- Holding prices constant, more people want to live there
- If more people move, rents increase
  - Could easily add other spillovers e.g., congestion, agglomeration
- So, not all people would move
- End up with new equilibrium where noone wants to change location

Spatial adjustment after a productivity shock



(e)



(f)

Outline

The spatial model in other settings

- Trade: interpret idiosyncratic shock as distribution of productivity for producing different types of good (Eaton and Kortum, 2002)
  - Under autarky, need to produce everything, even stuff with low productivity draw
  - Trade allows you to import things you aren't good at: increases average productivity in the economy
- Sorting on productivity
  - Idiosyncratic draw can be your productivity in a location (see, e.g, Lagakos and Waugh (2013); Bryan and Morten (2019))
  - Then get e.g., Roy model of sorting
- Include frictions (e.g., goods/people)
  - Only difference is one more term in the indirect utility function
  - No migration costs: $V_d = w_d - r_d + A_d$
  - Migration cost between o, d: $V_{od} = \underbrace{w_d - r_d + A_d}_{V_d} - c_{od}$

Outline

Rosen-Roback model: Exogenous prices

- 2 locations
- N locations

RR model: Endogenous prices

Extend to different settings

Empirical exercise

Wrap up

Outline for code to solve on the computeer

- Sally will go through this in TA section + provide simulations
  - If want to prep, can do so before Monday
- Choose N locations
  - Set amenity and productivity (get wage)
  - Set slope of housing supply function
  - Set total number of people in the economy
- Iterate on population
  - Guess initial π^0
  - Population $= N * \pi^0$
  - Rent $= f(N * \pi^0)$
  - Then, update $\pi^1 = f(r, w, A)$
  - How close are π^1 and π^0? Make an update $\pi^0 = 0.7 * \pi^0 + 0.3 * \pi^1$
  - Then repeat until π^1 and π^0 converge

Outline

Conclusions

- Basic spatial model is a versatile tool
  - Trade: within and across countries
  - Migration
  - Commuting
- Important equilibrium considerations:
  - Marginal, not average, migrant indifferent
  - Average welfare depends on both observed (wages, rents, amenities) and unobserved (idiosyncratic shock)
- Capitalization of productivity shocks into land depends on labor supply elasticity

References I

Bayer, P., Ferreira, F., and McMillan, R. (2007). A unified framework for measuring preferences for schools and neighborhoods. *Journal of Political Economy*, 115(4):588–638.

Bryan, G. and Morten, M. (2019). The Aggregate Productivity Effects of Internal Migration: Evidence from Indonesia. *Journal of Political Economy*, 127(5):2229–2268.

Eaton, J. and Kortum, S. (2002). Technology, Geography, and Trade. *Econometrica*, 70(5):1741–1779.

Lagakos, D. and Waugh, M. (2013). Selection, Agriculture, and Cross-Country Productivity Differences. *American Economic Review*, 103(2):948–980.

Moretti, E. (2011). Local Labor Markets. *Handbook of Labor Economics*.

Roback, J. (1982). Wages, Rents, and the Quality of Life. *The Journal of Political Economy*, 90(6):1257–1278.

Rosen, S. (1979). Wage-Based Indexes of Urban Quality of Life. In Mieszkowski, P. and Straszheim, M., editors, *Current Issues in Urban Economics*. Philadelphia.