TA Session:
Demographic Transition and Development
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Demographic Transition

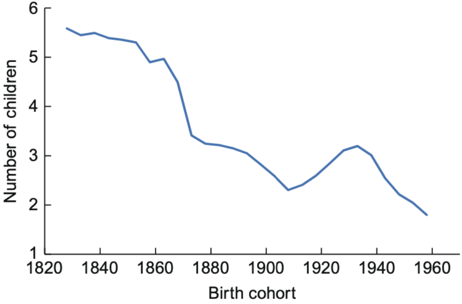

Fig. 1 Children ever born by cohort, United States (ie, average number of children for women born in a given year). *Jones, L.E., Tertilt, M., 2008. An economic history of the relationship between occupation and fertility—U.S. 1826–1960. In: Rupert, P. (Ed.), Frontiers of Family Economics, vol. 1. Emerald Group Publishing Limited, Bingley, UK (Table 1A).*

One-Parent Families

Modeling fertility choice: one-parent families

$$
\begin{aligned}
\max_{n,e,\ell} \quad & u(c) + \gamma^n u(n) + \gamma u(y') \\
\text{s.t.} \quad & c = A\ell H \\
& \ell + (\phi + e)n \leq 1 \\
& y = AH \\
& H' = (Be)^{\theta} H
\end{aligned}
$$

c: consumption

n: number of children chosen by the parent

γ^n: weight on the number of children, quantity

γ: weight attached to the welfare of the children, quality

ℓ: units of time to production

ϕ: units of time to raise a child

e: units of education time devoted to each child

θ: returns to education

H: human capital

A and B: technology parameters

Modeling fertility choice: one-parent families

Assume $\gamma^n > \gamma\theta$ and $u(\cdot) = \log(\cdot)$

$$\max \log(c) + \gamma^n \log(n) + \gamma\theta \log(e)$$

Replacing $c = A\ell H = A(1-(\phi+e)n)H$, and solving the FOC with respect to e and n, we obtain:

$$e^* = \frac{\gamma\theta}{\gamma^n - \gamma\theta}\phi$$
$$n^* = \frac{(\gamma^n - \gamma\theta)}{\phi\,(1+\gamma^n)}$$

$\rightarrow$ an increase of θ over time decreases fertility rates n.

The demographic transition can be explained by increasing returns to human capital (quantity-quality trade-off).

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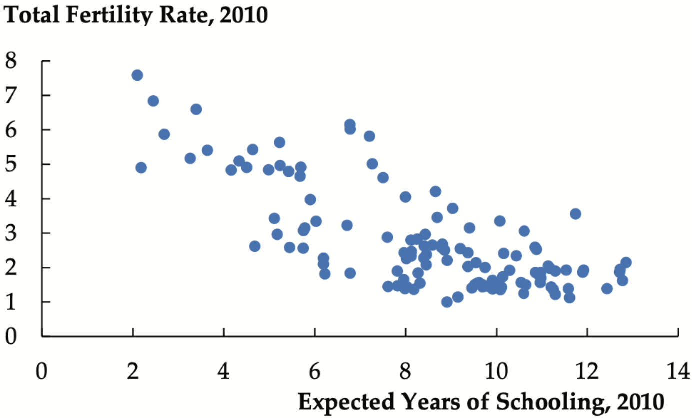
The demographic transition can be explained by increasing returns to human capital (quantity-quality trade-off).

$\rightarrow$ the return to human capital θ enters **positively** into the optimal education choice and **negatively** into the optimal fertility choice.

Data: Fertility and Schooling



Source: Doepke and Tertilt (2016)

Modeling fertility choice: one-parent families

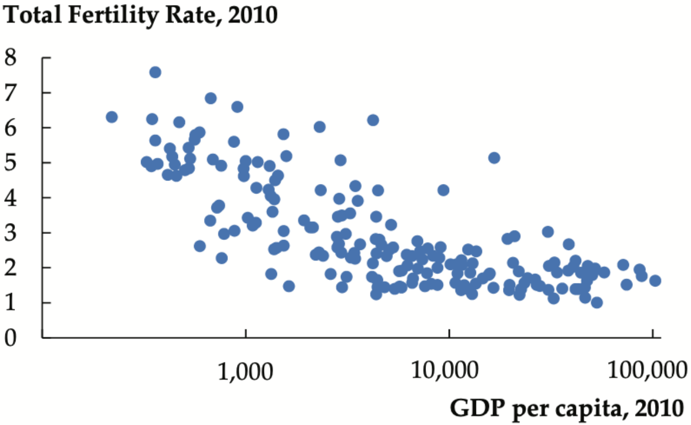
$$e^* = \frac{\gamma\theta}{\gamma^n - \gamma\theta}\phi$$
$$n^* = \frac{(\gamma^n - \gamma\theta)}{\phi\,(1 + \gamma^n)}$$

Replacing e^* into the human capital production function, we find the equilibrium growth rate:

$$\frac{H'}{H} = \left(B\frac{\gamma\theta\phi}{\gamma^n - \gamma\theta}\right)^{\theta}$$

$\rightarrow$ low cost of children ϕ and low returns to education θ lead to **high** number of children n^* and also a **low** growth rate $\frac{H'}{H}$.

Data: Fertility and GDP per Capita



Source: Doepke and Tertilt (2016)

Two-Parent Families

Modeling education choice: two-parent families

Overview of the model

- Families consist of a husband, a wife, a son, and a daughter.
- Men and women are not perfect substitutes in market production.
- Men and women disagree about how much they care about their children's well-being
- The couple solves a Pareto problem with fixed bargaining weights
- All consumption in families is public.
- Only women raise children, men work full time

Modeling education choice: two-parent families

$$\max_{e_f, e_m, c} \lambda_f \left[u(c) + \gamma_f u\left(y'\right)\right] + \left(1 - \lambda_f\right)\left[u(c) + \gamma_m u\left(y'\right)\right]$$

$$\begin{aligned}
\text{s.t. } & c = A\left(\ell_f H_f\right)^{\alpha} H_m^{1-\alpha} \\
& \ell_f + e_f + e_m \leq 1 \\
& H_f' = \left(B e_f\right)^{\theta} H_f^{\beta} H_m^{1-\beta} \\
& H_m' = \left(B e_m\right)^{\theta} H_f^{\beta} H_m^{1-\beta} \\
& y' = A\left(H_f'\right)^{\alpha}\left(H_m'\right)^{1-\alpha}
\end{aligned}$$

c: household consumption, θ: returns to education

λ_f, λ_m: bargaining weight of the woman f, of the man m

γ_f, γ_m: altruism parameter of the woman f, of the man m

ℓ_f: units of time to production

e_f, e_m: units of education time devoted to her daughter f and her son m.

β: the relative importance of women vs men in transmitting HK

α: the relative importance of women vs men in production

Modeling education choice: two-parent families

Assume $u(\cdot) = \log(\cdot)$

$$\max \log(c) + \delta \left[\alpha\theta \log\left(e_f\right) + (1-\alpha)\theta \log\left(e_m\right)\right]$$

with $\delta \equiv \lambda_f \gamma_f + (1-\lambda_f)\,\gamma_m$

Replacing $c = A\ell H = A(1 - e_f - e_m)H$, and solving the FOC with respect to e_f and e_m, we obtain:

$$e_m^* = \frac{(1-\alpha)\delta\theta}{\alpha + \delta\theta}$$

$$e_f^* = \frac{\alpha\delta\theta}{\alpha + \delta\theta}$$

$$\frac{e_f^*}{e_m^*} = \frac{\alpha}{(1-\alpha)}$$

$\rightarrow$ The more productive women are in production (higher α), the smaller is the gender education gap. Higher female wage increase the opportunity cost of time and hence make children more costly.

Modeling education choice: two-parent families

Plugging the ratio of HK $(\frac{H'_f}{H'_m} = (\frac{e^*_f}{e^*_m})^\theta)$ back into the HK production function, we get the equilibrium growth rate:

$$\frac{H'_f}{H_f} = \frac{H'_m}{H_m} = \frac{H'}{H} = B^\theta \left(e_m\right)^{(1-\beta)\theta} \left(e_f\right)^{\theta\beta} = \left\{ \frac{B\delta\theta}{\alpha + \delta\theta} (1-\alpha)^{1-\beta} \alpha^\beta \right\}^\theta$$

with $\delta \equiv \lambda_f \gamma_f + (1 - \lambda_f)\, \gamma_m$

$\rightarrow$ if $\gamma_f > \gamma_m$, the growth increases in female bargaining power λ_f.

The Family as Driver of Political Change

Voting for Women's Rights and Fertility Choice

Model overview

- Overlapping generations of married men and women.
- Families consist of a husband, a wife, and an equal number of sons and daughters n.
- Decisions about fertility, the education of their children, and the allocation of consumption between the husband and the wife.
- Relative bargaining power of the wife represents women's rights.
- Women's rights are endogenous.

Voting for Women's Rights and Fertility Choice

Suppose $\gamma_f > \gamma_m$. For $i \in \{m, f\}$

$$U_i\left(c_i, c_{-i}, n, U'_m, U'_f\right) = u\left(c_i, c_{-i}, n\right) + \gamma_i \left(\frac{U'_m + U'_f}{2}\right)$$

where:

$$u\left(c_i, c_{-i}, n\right) = \log\left(c_i\right) + \sigma \log\left(c_{-i}\right) + \delta \log(n)$$

U'_m, U'_f: average of the utilities of their sons, daughters

$0 < \sigma < 1$: weight on spousal consumption

$\delta > 0$: weight on the number of children.

$\gamma_i > 0$: weight attached to the welfare of the children

$\rightarrow$ men and women have different views of the quantity-quality trade-off

Voting for Women's Rights and Fertility Choice

Time constraint:

$$t_f + \left(\phi + e^f + e^m\right) n \leq 1$$
$$t_m \leq 1$$

Home production function:

$$c_m + c_f = A\left(t_f H_f\right)^{\alpha}\left(t_m H_m\right)^{1-\alpha}$$

Accumulation of human capital:

$$H_f' = \max\left\{1, \left(Be^f\right)^{\theta} H_f^{\beta} H_m^{1-\beta}\right\}$$
$$H_m' = \max\left\{1, \left(Be^m\right)^{\theta} H_f^{\beta} H_m^{1-\beta}\right\}$$

$\rightarrow$ If the education technology is relatively unproductive (i.e., B or θ is low) $\rightarrow$ corner solution: parents do not educate their children ($e_f = 0, e_m = 0$).

Voting for Women's Rights and Fertility Choice

Patriarchy regime: Men make decisions, women obey.

$$\{c_m, c_f, n, e_m, e_f\} = \text{argmax}\left\{U_m\left(c_m, c_f, n, U'_m, U'_f\right)\right\}$$

Empowerment regime: Equal power and efficient bargaining.

$$\{c_m, c_f, n, e_m, e_f\} = \text{argmax}\left\{\frac{U_m\left(c_m, c_f, n, U'_m, U'_f\right) + U_f\left(c_f, c_m, n, U'_m, U'_f\right)}{2}\right\}$$

Men vote on regime (affects current and future marriages).

Voting for Women's Rights and Fertility Choice

Why would the empowerment regime be chosen?

1. More consumption for daughters (always present)
2. "Time inconsistent preferences" (only with education)
   - Men disagree with their son-in-law about optimal resource allocation across generations.
   - More power for daughters solves this problem
3. Human capital externality (only with education)
   - Positive effect of education on children's spouses.
   - Leads to underinvestment in human capital of future sons/daughters in-law.
   - More power for all mothers mitigates this problem.

Voting for Women's Rights and Fertility Choice

Low return to education (θ low)

- Parents dont educate, and decision problem is static.
- Political regime only affects consumption share of husbands and wives.
- Mens incentives for sharing power are low. Only (1)
  - Men also value the utility of their daughters $\rightarrow$ taste for equality in the future.
  - Sharing if γ_m high enough (care sufficiently much about their daughter) or σ low enough (low utility for daughters, granddaughters etc.)

Voting for Women's Rights and Fertility Choice

High return to education (θ high)

- Dynasty accumulates human capital.
- Political regime affects speed of accumulation.
  - All variables grow at rate $\left(Be_f^{\beta} e_m^{1-\beta}\right)^{\theta}$
- For sufficiently high return, men prefer to share power.
  - (2) "Time inconsistent preferences"

$$U_m = u_m + \gamma_m \left(\frac{U'_m + U'_f}{2}\right)$$
$$= u_m + \gamma_m \left(\frac{1}{2}\left[u'_m + \gamma_m \left(\frac{U''_m + U''_f}{2}\right)\right] + \frac{1}{2}\left[u_f + \gamma_f \left(\frac{U''_m + U''_f}{2}\right)\right]\right)$$

Voting for Women's Rights and Fertility Choice

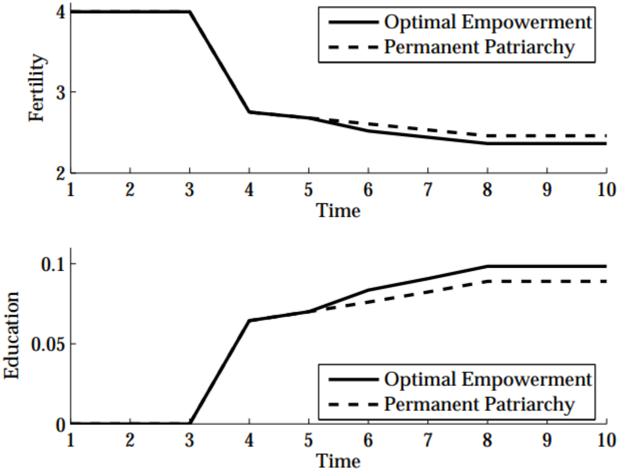


Figure I: Fertility Rate and Female Education in Numerical Example

Voting for Women's Rights and Fertility Choice

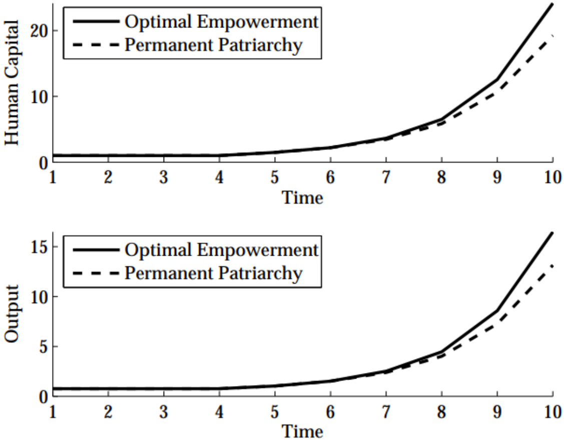
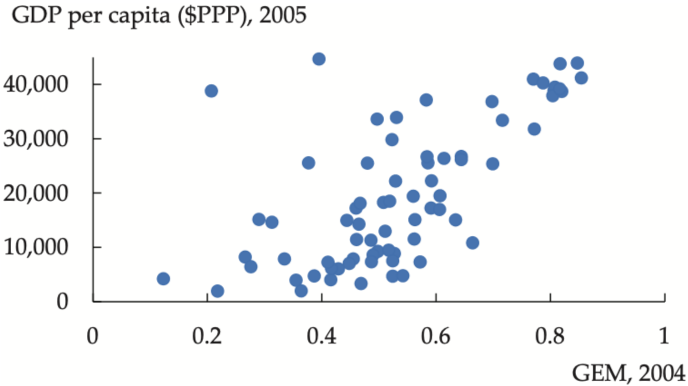


Figure II: Female Human Capital and Output per Adult in Numerical Example

Data: Women's Rights and GDP per Capita



Source: Doepke and Tertilt (2016)

Voting for Children's Rights

Model overview

- Two types of workers: NS skilled and NU unskilled workers.
- Each worker has n children
- Only the children of the unskilled workers are working.
- A working child supplies λ units of unskilled labor.
- Two regimes: *laissez faire* and *ban*.
- Competitive production. Production technology: $Y = AX_S^{\alpha} X_U^{1-\alpha}$

Voting for Children's Rights

Labor supply:

$$X_S^{\text{laissez faire}} = N_S \qquad X_S^{\text{ban}} = N_S$$
$$X_U^{\text{laissez faire}} = N_U + \lambda n N_U \qquad X_U^{\text{ban}} = N_U$$

Wages:

$$w_S^{\text{laissez faire}} = A\alpha\left(\frac{(1+\lambda n)N_U}{N_S}\right)^{1-\alpha} \qquad w_S^{\text{ban}} = A\alpha\left(\frac{N_U}{N_S}\right)^{1-\alpha}$$
$$w_U^{\text{laissez faire}} = A(1-\alpha)\left(\frac{N_S}{(1+\lambda n)N_U}\right)^{\alpha} \qquad w_U^{\text{ban}} = A(1-\alpha)\left(\frac{N_S}{N_U}\right)^{\alpha}$$

The **ratios of wages** under the two policies are:

$$\frac{w_S^{\text{ban}}}{w_S^{\text{laissez faire}}} = \left(\frac{1}{1+\lambda n}\right)^{1-\alpha} < 1$$
$$\frac{w_U^{\text{ban}}}{w_U^{\text{laissez faire}}} = (1+\lambda n)^{\alpha} > 1$$

$\rightarrow$ unskilled workers may be in favor of banning child labor?

Voting for Children's Rights

Family income:

$$I_S^{\text{laissez faire}} = w_S^{\text{laissez faire}} \qquad I_S^{\text{ban}} = w_S^{\text{ban}}$$
$$I_U^{\text{laissez faire}} = (1 + \lambda n) w_U^{\text{laissez faire}} \qquad I_U^{\text{ban}} = w_U^{\text{ban}}$$

The **income ratios** are:

$$\frac{I_S^{\text{ban}}}{I_S^{\text{liissez faire}}} = \left(\frac{1}{1+\lambda n}\right)^{1-\alpha} < 1$$
$$\frac{I_U^{\text{ban}}}{I_U^{\text{lassez faire}}} = \left(\frac{1}{1+\lambda n}\right)^{1-\alpha} < 1$$

$\rightarrow$ income falls for both groups $\rightarrow$ public support for introducing child-labor restrictions should be low.

Voting for Children's Rights

- Schooling: only a fraction $1-s$ of unskilled workers has working children.

Wages:

$$w_S^{\text{laissez faire}} = A\alpha\left(\frac{(1+\lambda(1-s)n)N_U}{N_S}\right)^{1-\alpha} \qquad w_S^{\text{ban}} = A\alpha\left(\frac{N_U}{N_S}\right)^{1-\alpha}$$

$$w_U^{\text{laissez faire}} = A(1-\alpha)\left(\frac{N_S}{(1+\lambda(1-s)n)N_U}\right)^{\alpha} \qquad w_U^{\text{ban}} = A(1-\alpha)\left(\frac{N_S}{N_U}\right)^{\alpha}$$

Family income:

$$\begin{aligned}
I_S^{\text{laissez faire}} &= w_S^{\text{laissez faire}} & > \quad & I_S^{\text{ban}} = w_S^{\text{ban}} \\
I_{U_{(1-s)}}^{\text{laissez faire}} &= (1+\lambda n)w_U^{\text{laissez faire}} & > \quad & I_{U_{(1-s)}}^{\text{ban}} = w_U^{\text{ban}} \\
I_{U_{(s)}}^{\text{laissez faire}} &= w_U^{\text{laissez faire}} & < \quad & I_{U_{(s)}}^{\text{ban}} = w_U^{\text{ban}}
\end{aligned}$$

$\rightarrow$ technological change $\uparrow$ the demand for HK $\rightarrow$ families educate their children $\rightarrow$ become supporters of a child labor ban

Voting for Children's Rights

- Ban is in place. Should the ban be abandoned?
- Fertility: a fraction ν choose ex-ante $n^{\text{b}} < n$ (children more expensive); $(1-\nu)$ choose the number of children ex-post.

Wages:

$$w_S^{\text{ban}} = A\alpha \left(\frac{N_U}{N_S}\right)^{1-\alpha} \qquad w_S^{\text{laissez faire}} = A\alpha \left(\frac{(1+\lambda(\nu n^{\text{b}} - (1-\nu)n))N_U}{N_S}\right)^{1-\alpha}$$

$$w_U^{\text{ban}} = A(1-\alpha)\left(\frac{N_S}{N_U}\right)^{\alpha} \qquad w_U^{\text{laissez faire}} = A(1-\alpha)\left(\frac{N_S}{(1+\lambda(\nu n^{\text{b}} - (1-\nu)n))N_U}\right)^{\alpha}$$

Family income:

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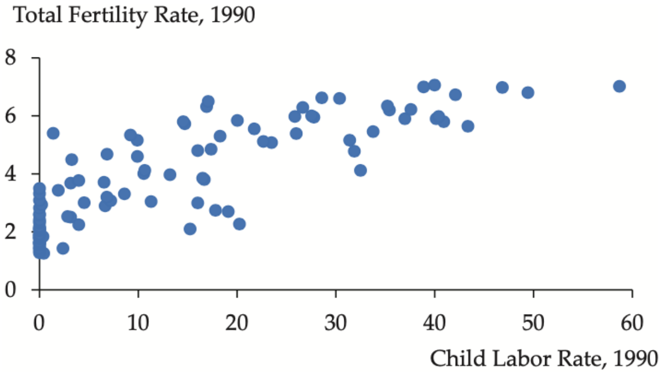
I_S^{\text{laissez faire}} &= w_S^{\text{laissez faire}} & > \quad & I_S^{\text{ban}} = w_S^{\text{ban}} \\
I_{U_{(1-\nu)}}^{\text{laissez faire}} &= (1+\lambda n) w_U^{\text{laissez faire}} & > \quad & I_{U_{(1-\nu)}}^{\text{ban}} = w_U^{\text{ban}} \\
I_{U_{(\nu)}}^{\text{laissez faire}} &= (1+\lambda n^b) w_U^{\text{laissez faire}} & < \quad & I_{U_{(\nu)}}^{\text{ban}} = w_U^{\text{ban}} \quad \text{if } \frac{1+\lambda n^{\text{b}}}{\left(1+\lambda\left(\nu n^b + (1-\nu)n\right)\right)^{\alpha}} < 1
\end{aligned}$$

$\rightarrow$ policy persistence: Once a policy is in place, families make decisions that in the future increase political support for maintaining the policy.

Data: Child Labor and Fertility Rate



Source: Doepke and Tertilt (2016)

Thank you!