Risk Sharing Tests and Covariate Shocks

Ethan Ligon

January 9, 2026

Introduction

How do Shocks Affect Welfare?

Some shock affects a household's income or assets. E.g.,

- A cow dies;
- Hail destroys a crop of tomatoes;
- A family's shop is looted;
- Flooding makes a field unplantable;
- Drought withers crops.

Shocks to income & assets need not affect welfare

- (Informal?) markets for credit & insurance support "risk-sharing"
- "Covariate" shocks may be harder to insure
- Integration of markets becomes key question

Pareto optimal allocations—all risk is shared

Townsend-style Risk-sharing Test

Townsend (1994) devised a very simple and influential test of the null hypothesis that consumption allocations over dates & states are Pareto efficient, using data from three small Indian villages.

- The test takes the form of a simple two-way fixed effects (TWFE) panel regression, with (log) total expenditures as the dependent variable, and some "shock" as an independent variable.
- IF the coefficient on the "shock" enters significantly, we reject full risk-sharing.

For Example...

Townsend-style risk-sharing regression; panel estimation with two-way fixed effects (TWFE):

$$\log \text{Cons. Exp}_{it} = \alpha_i + \eta_t + \beta \text{Shock}_{it} + \epsilon_{it}$$

Townsend (1994)	India	Income	0.11^{**}
Gertler and Gruber (2002)	Indonesia	ADL	0.195^{**}
Angelucci and De Giorgi (2009)	Mexico	Progresa	$+^{***}$
Jack and Suri (2014)	Kenya	"Negative shock"	0.27
		"Health shock"	0.17
Karlan et al. (2014)	Ghana	Index insurance	$+$
Banerjee et al. (2015)	India	Microfinance	$+$
Haushofer and Shapiro (2016)	Kenya	Cash Transfers	0.18^{***}
Angelucci et al. (2018)	Mexico	Progresa	$+^{***}$
Santàeulalia-Llopis and Zheng (2018)	China	Income	$+^{***}$
Kinnan (2022)	Thailand	Income	0.07^{***}

A Puzzle

Consider this Townsend-style risk-sharing regression
Panel estimation with two-way fixed effects (TWFE) in Uganda:

$$\log x_{it} = \alpha_i + \eta_t + \beta \text{Shock}_{it} + \epsilon_{it}$$

Income	Drought	Floods	Pests
0.075^{***}	0.043^{***}	0.091^{***}	0.094^{***}
(0.006)	(0.011)	(0.027)	(0.024)

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(0.006)	(0.011)	(0.027)	(0.024)

What?!

- Income has expected sign; but
- Big negative covariate shocks improve welfare?!

Why?!

Obvious explanation from Econ 101:

- Covariate shocks are a local *supply* shock, and (*ceteris paribus*) increase *prices*.
- Some goods have inelastic demands (i.e., elasticity less than 1), which implies the elasticity of expenditures w.r.t. prices *greater* than one for these goods.
- Most observed expenditures are food, which Engel tells us should have inelastic demand.

Puzzle Solved?

- But why does Townsend regression mislead?
- And can we construct risk-sharing tests against covariate shocks?

Model

Model of Marginal Utility of Expenditures

Theory says if allocations efficient then for every state s

$$-\log \lambda_{it}(s) = \log \theta_i - \log \mu_t(s),$$

where:

- θ_i Household i's Pareto weight;
- $\mu_t(s)$ The (shadow) price of aggregate resources at time t; and
- $\lambda_{it}(s)$ The marginal utility of expenditures.

Testing Efficiency (λ observed)

Suppose panel data on realizations for households over time, but observe $\log \lambda_{it}$ only (with classical measurement error). Then we have

$$-\log \lambda_{it}(s) = \log \theta_i - \log \mu_t + \epsilon_{it}.$$

In reduced form, we replace $-\log \mu_t$ and $\log \theta_i$ with latent variables,

$$-\log \lambda_{it} = a_i + b_t + \epsilon_{it}.$$

Since income or other shocks don't enter this equation, we can test for efficiency using an exclusion restriction

$$-\log \lambda_{it}(s) = a_i + b_t + d\mathsf{Shock}_{it} + \epsilon_{it};$$

i.e., under the null of full insurance $d = 0$.

More on λ

This object λ shows up any time we work with models involving time or uncertainty (here we assume intertemporally separable utility):

Euler Equation $\beta^j \, \mathrm{E}_t \, R_{t+j} \frac{\lambda_{it+j}}{\lambda_{it}} = 1$

Money Flexibility $\omega = -\frac{\partial \log \lambda}{\partial \log x}$

Relative Risk Aversion $\text{RRA} = -\frac{\partial \log \lambda}{\partial \log x} + f(U)x$

Intertemporal Elasticity of Substitution $\text{IES} = -\text{RRA}^{-1}$

Constant Frisch Elasticity Demands

Revealing λ using *expenditures*

We don't observe λ directly, but we have data on consumer expenditures.

- We know in *principle* that we can infer λ by observing consumer choices of quantities at different prices (e.g., Afriat's Theorem); this also means identifying corresponding ordinal preferences (i.e., indifference curves) and demand functions.
- When can we infer λ from (item level) consumption expenditures?

Class of Candidate Utility Functions

Let $\mathcal{U}_n$ be the set of strictly increasing, strictly concave, twice-continuously differentiable functions mapping $\mathbb{R}^n_+$ into $\mathbb{R}$, and call $\mathcal{U}_n$ the set of *regular* utility functions over $\mathbb{R}^n_+$.

Consumer's Problem
Assume household has a regular utility function $U \in \mathcal{U}_n$, and solves

$$\max_{c \in \mathbb{R}^n_+} U(c) \qquad \text{s.t. } p'c \leq x.$$

This leads to FOC

$$u_j(c) = \lambda p_j, \qquad j = 1, 2, \ldots, n$$

where $u_j = \partial U / \partial c_j$. Putting in vector form and inverting yields "Frischian" demands

$$c(\lambda p) = u^{-1}(\lambda p) = f(\lambda p).$$

Some Properties of Frischian Demands

Remark

1. *The demand system* f *is continuously differentiable.*
2. *The matrix* $F = [f_{ij}]$ *is symmetric and negative definite.*
3. *Demands* f *are strictly decreasing in* λ*.*
4. $f_{ii} < 0$*.*
5. $f(p, \lambda)$ *is homogeneous of degree zero in* $(p, 1/\lambda)$*.*
6. *Frisch expenditures* $x_j(p, \lambda) \equiv p_j f_j(p, \lambda)$ *are linearly homogeneous in* $(p, 1/\lambda)$*.*

We're particularly interested in (6), the homogeneity of expenditures.

Constructing λs from Expenditures

The fundamental risk-sharing equation is

$$\log \lambda = \log \mu(p) - \log \theta,$$

but we don't directly observe λ, only expenditures.

Challenge
To preserve the risk-sharing test we need to be able to write some transformation of expenditures as an additively separable function of prices and $\log \lambda$.

CRRA
CRRA utility does this by summing up all expenditures, yielding

$$\log x = \frac{-1}{\gamma} \log \lambda + \log \pi(p)$$

for some linearly homogeneous price index $\pi(p)$.

What other regular preferences work?

Expenditures λ-separable

Condition
The Frischian expenditures on good j, $x_j(p, \lambda) \equiv p_j f_j(p\lambda)$ are λ-separable if there exist functions (ϕ_j, a_j, b_j) such that

$$\phi_j(x_j(p, \lambda)) = a_j(p) + b_j(\lambda), \tag{1}$$

with ϕ_j continuously differentiable and a_j either non-constant or zero.

Note that while rationalizability is a property of the entire system of demands and expenditures, λ-separability is a property of a particular good. In particular it's possible that some but not all demands or expenditures are λ-separable.

Demands and utilities when expenditures are λ-separable

Exploiting the fact that expenditures must be linearly homogeneous, it turns out that one can write any rationalizable λ-separable expenditures in the form

$$k(p+\lambda) = g(\lambda)\ell(p) + h(p),$$

which is called the *generalized Pexider* equation. This gives us a single functional equation in two variables, which can be solved for the four functions g, h, k, and ℓ.

- Exploiting this allows us to describe all rationalizable demands and utilities when expenditures are λ-separable.

Expenditures that are λ-separable

Theorem

If expenditures for some good j are consistent with the consumer's FOC and expenditures are λ-separable with ϕ_j increasing; $a_j(p)$ either non-constant or zero, and continuous at a point; and with b_j continuous at a point, then transformation functions ϕ_j, Frischian demands f_j and rationalizing marginal utility u_j must satisfy one of the following two cases for positive constants α_j, β_j, and σ_j:

1. *(Constant Frisch Elasticity):* $\phi_j(x_j) = \log(x_j)$; $f_j(p, \lambda) = (\alpha_j/(\lambda p_j))^{\beta_j}$; *and* $u_j(c) = \alpha_j c_j^{-1/\beta_j}$.
2. *(Generalized Stone-Geary):* $\phi_j(x_j) = x_j^{\sigma_j}$; $f_j(p, \lambda) = [(\beta_j/(\lambda p_j))^{\sigma_j} + \alpha_j]^{1/\sigma_j}$; *and* $u_j(c) = \beta_j \left(c_j^{\sigma_j} - \alpha_j\right)^{-1/\sigma_j}$.

λ-separable demands that can be used in the risk-sharing equation

Only the Constant Frisch Elasticity (CFE) form lends itself to being easily used in the risk-sharing equation; we obtain

$$\log(x_j) = a_j(p) + \beta_j \log \lambda,$$

or after allowing for household characteristics d (which we haven't discussed, but which is a strength of CFE),

$$\log(x_j) = a_j(p) + \gamma_j(d) + \beta_j \log \lambda + \epsilon_j.$$

NB: $\log \lambda$ common across all equations
So: we don't need to observe *all* expenditures to estimate λ, but can pick and choose.

CFE Elasticities

Frisch (1959) described a demand system for n goods that could be completely characterized by n elasticities:

Frisch elasticities These are the "Frisch" elasticities β_j, $j = 1, \ldots, n$, one for each good.

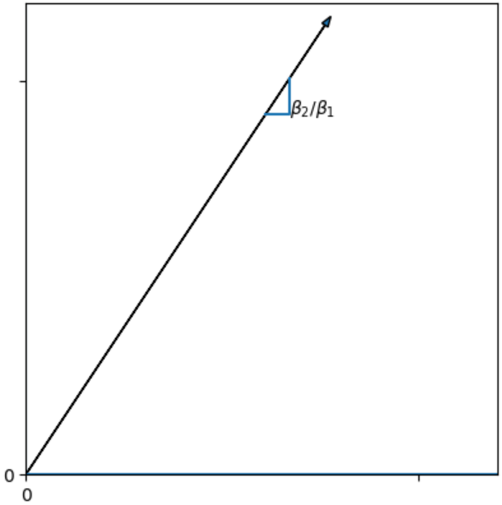
"Money Flexibility" This is Frisch's name for the elasticity of λ w.r.t. total expenditures:

$$\omega = -\frac{\partial \log \lambda}{\partial \log x} = \frac{x}{\sum_j \beta_j x_j}.$$

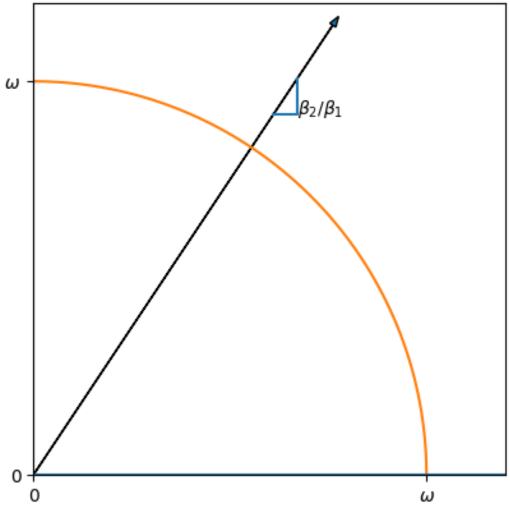
Adding up That's $n + 1$ elasticities, but one is pinned down by the budget constraint, leaving exactly n degrees of freedom.

In the CFE demand system the Frisch elasticities are fixed parameters, but ω is a function of prices and income.

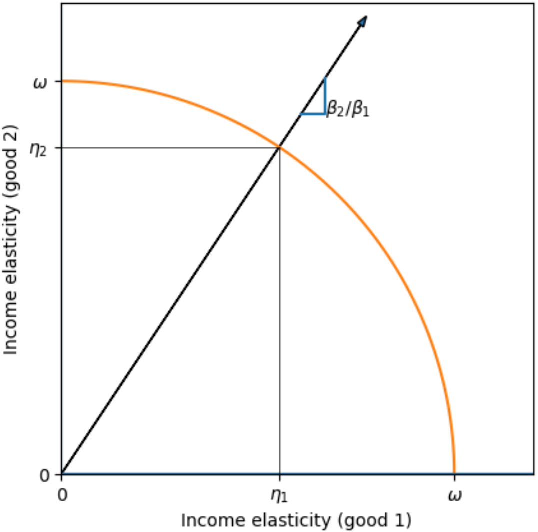
CFE Elasticities



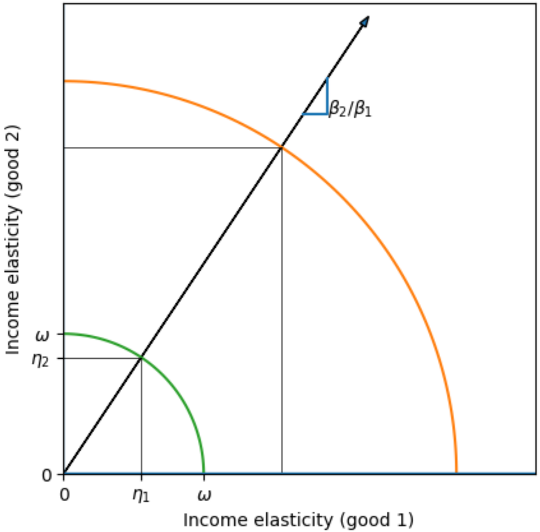
CFE Elasticities



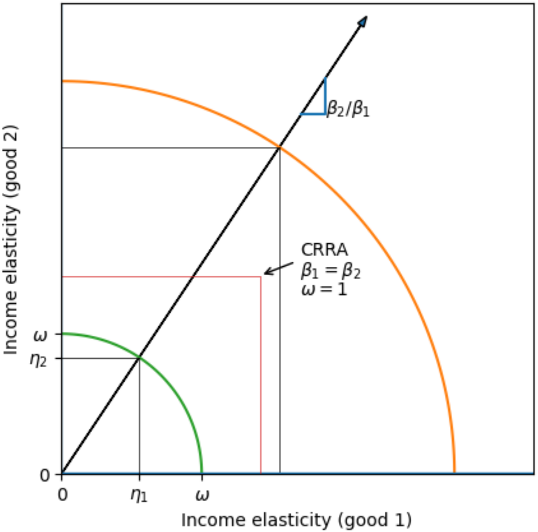
CFE Elasticities



CFE Elasticities



CFE Elasticities



Estimating CFE Demands

Estimating β from *composition* of expenditures

Consider a random household having CFE demands, with characteristics d facing prices p.

- Let Y be the vector of log consumption expenditures;
- Let $\mu(p,d) = \mathrm{E}(Y|p,d)$.

Then the conditional covariance matrix of log expenditures is given by

$$\mathrm{E}((Y-\mu(p,d))(Y-\mu(p,d))^\top|p,d) = \mathsf{Var}(\log\lambda)\beta\beta^\top. \qquad (2)$$

Thus, the (conditional) covariance matrix of log expenditures is rank one and identifies the vector of elasticities β up to the variance of the scalar $\log\lambda$. Normalizing this last variance to one is harmless. Details on estimating the conditional covariance matrix given in Ligon (2019).

Estimating $w = -\log \lambda$ using generated $\hat{\beta}$

Regress expenditures on $\hat{\beta}$ using cross-sectional data on households $i = 1, \ldots, N$:

$$\log(x_{ji}) = \alpha_j(p) + \gamma_j(d_i) + \hat{\beta}_j w_i + \epsilon_{ji},$$

obtaining estimates $\hat{w}_i$.

Data

LSMS data from Uganda – Households

Empirical illustration today using LSMS data from Uganda. This forms a eight year unbalanced panel with a total of 22,500 usable observations.

Round	2005	2009	2010	2011	2013	2015	2018	2019
2005-06	3122	2606	2386	2363	1566	1470	1356	1288
2009-10	—	2974	2617	2581	1685	1578	1451	1376
2010-11	—	—	2685	2487	1603	1506	1389	1322
2011-12	—	—	—	2843	1712	1610	1481	1410
2013-14	—	—	—	—	3117	2870	2613	2468
2015-16	—	—	—	—	—	3305	2968	2796
2018-19	—	—	—	—	—	—	3241	3007
2019-20	—	—	—	—	—	—	—	3076

Table 2: Households' Attrition in Uganda LSMS

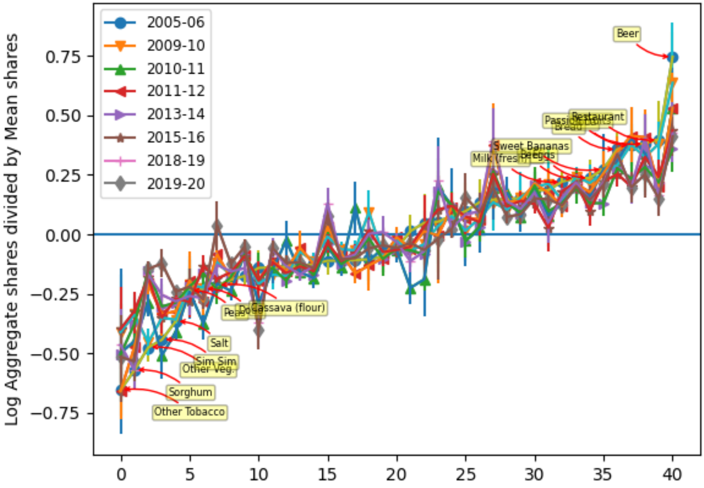
Data from Uganda — Household characteristics

(t/N_t)	Girls	Boys	Women	Men	Rural	log HSize
2005-06	1.55	1.56	1.17	1.05	0.72	1.48
3122	(1.48)	(1.49)	(0.81)	(0.82)	(0.45)	(0.68)
2009-10	1.73	1.77	1.26	1.13	0.74	1.60
2974	(1.53)	(1.59)	(0.82)	(0.90)	(0.44)	(0.64)
2010-11	1.85	1.86	1.31	1.16	0.78	1.66
2685	(1.60)	(1.62)	(0.88)	(0.93)	(0.41)	(0.63)
2011-12	1.79	1.77	1.29	1.15	0.79	1.63
2843	(1.57)	(1.58)	(0.80)	(0.90)	(0.40)	(0.62)
2013-14	1.58	1.59	1.25	1.12	0.74	1.54
3117	(1.50)	(1.50)	(0.77)	(0.86)	(0.44)	(0.64)
2015-16	1.28	1.29	1.17	1.05	0.75	1.34
3304	(1.39)	(1.40)	(0.77)	(0.88)	(0.43)	(0.74)
2018-19	1.46	1.42	1.23	1.11	0.75	1.48
3176	(1.40)	(1.36)	(0.72)	(0.86)	(0.44)	(0.63)
2019-20	1.42	1.37	1.23	1.09	0.76	1.46
3076	(1.40)	(1.32)	(0.71)	(0.85)	(0.43)	(0.64)
Pooled	1.57	1.57	1.23	1.11	0.75	1.52
24297	(1.49)	(1.49)	(0.79)	(0.87)	(0.43)	(0.66)

Figures in parentheses are standard deviations.

Table 3: Mean characteristics of households by year.

Data from Uganda —Consumption Expenditures



Data from Uganda—Incidence of Shocks

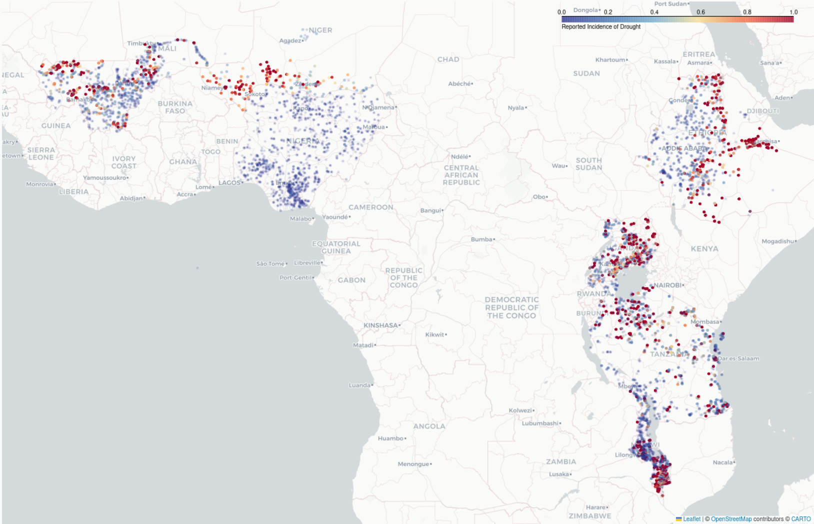
Shock	2005	2009	2010	2011	2013	2015	2018	2019	Total
Health	82	377	301	156	133	88	190	197	1524
Theft	349	233	96	55	76	62	75	83	1029
Death	423	74	58	35	66	35	44	49	784
Death of earner	99	27	17	19	30	19	20	19	250
Drought	1234	1344	710	560	914	598	736	529	6625
Floods	426	61	102	148	98	62	74	117	1088
Pests	475	219	77	92	71	53	130	84	1201
Prices	71	113	54	65	67	12	78	29	489

Table 4: Reported incidence of different kinds of shocks by year.

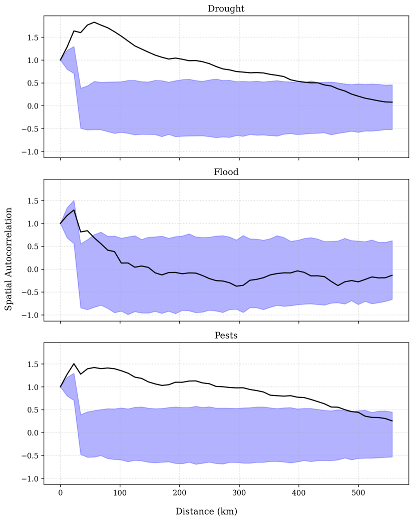
Data from Uganda—Takeaways re: Shocks

- Drought is by far the most frequently reported shock.
- Meteorologists regard 2005–08 as a period of major drought for Uganda, with 2010-11 and 2014-15 periods of minor drought (Byakatonda et al., 2021).
- Health shocks (both illness and accidents) are the second most frequently reported shocks.
- Pests 3rd most frequent (includes both crop pests and livestock disease).
- Other shocks: floods, death, theft, and adverse changes in agricultural prices.

Reports of Drought in Africa (and the spread of the LSMS-ISA)

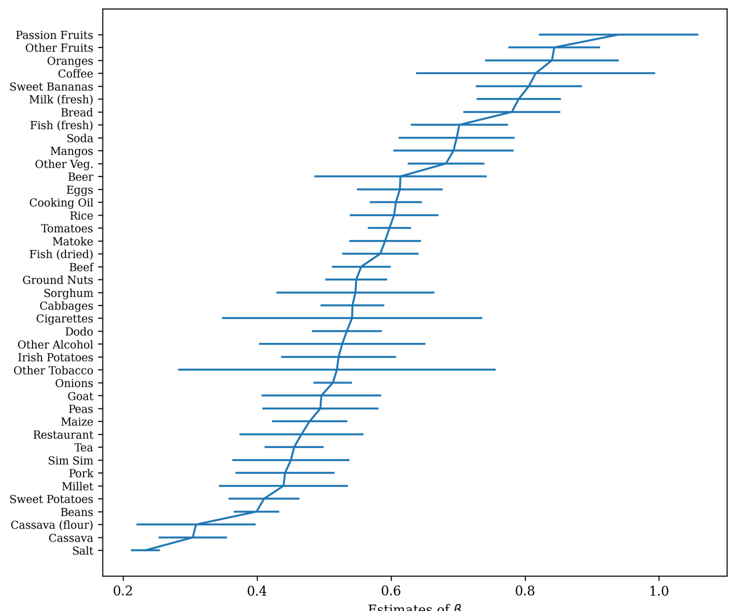


Spatial autocorrelation of reported shocks

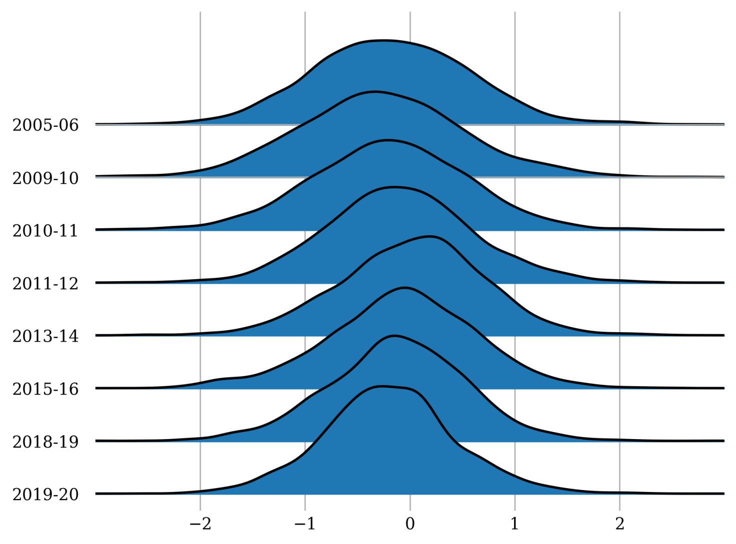


Results from Demand Estimation

Frisch Elasticities

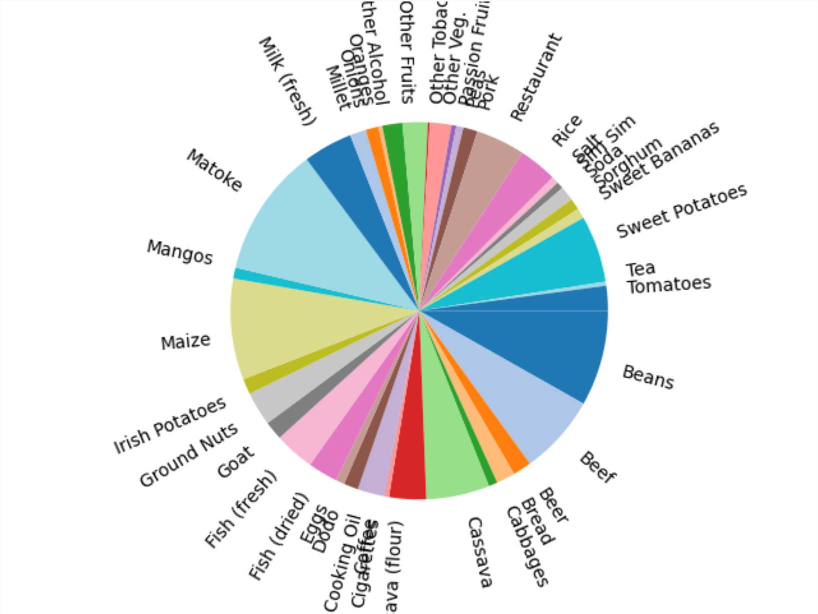


$w = -\log \lambda$

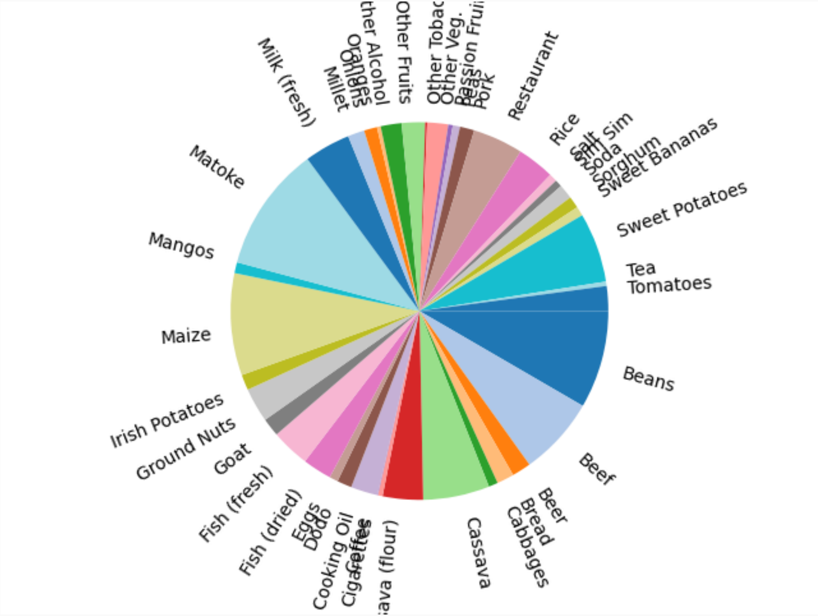


2005-06
2009-10
2010-11
2011-12
2013-14
2015-16
2018-19
2019-20
−2
−1
0
1
2

Engel Pie



Milk (fresh)
Millet
Onions
Oranges
ther Alcohol
Other Fruits
Other Tobac
Other Veg.
Passion Frui
Peas
Pork
Restaurant
Rice
Salt
Sim Sim
Soda
Sorghum
Sweet Bananas
Sweet Potatoes
Tea
Tomatoes
Beans
Beef
Beer
Bread
Cabbages
Cassava
ava (flour)
Cooking Oil
Cigarettes
Coffee
Dodo
Eggs
Fish (dried)
Fish (fresh)
Goat
Ground Nuts
Irish Potatoes
Maize
Mangos
Matoke


Milk (fresh)
Millet
Oranges
Other Fruits
Other Veg.
Pork
Restaurant
Rice
Salt
Soda
Sorghum
Sweet Bananas
Sweet Potatoes
Tea
Tomatoes
Beans
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Fish (dried)
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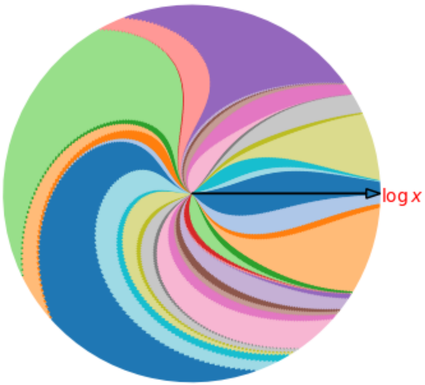
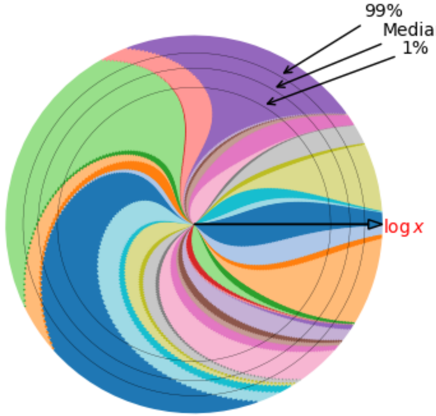
Maize
Mangos
Matoke

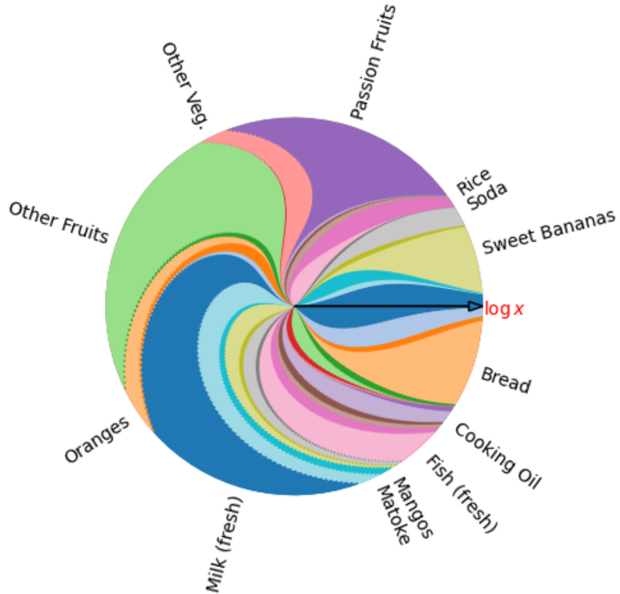
Engel Pie



Engel Pie

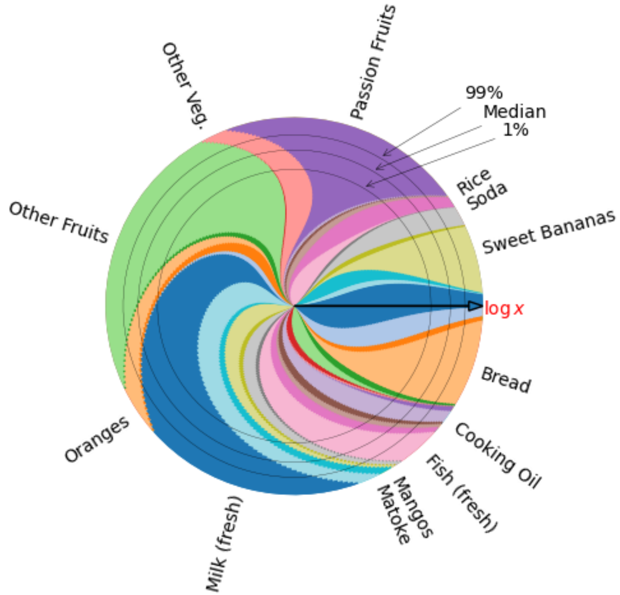


Engel Pie



Passion Fruits
Other Veg.
Rice
Soda
Sweet Bananas
Other Fruits
log x
Bread
Oranges
Cooking Oil
Fish (fresh)
Mangos
Matoke
Milk (fresh)

Engel Pie



Passion Fruits
Other Veg.
99%
Median
1%
Rice
Soda
Sweet Bananas
Other Fruits
log x
Bread
Cooking Oil
Oranges
Fish (fresh)
Mangos
Matoke
Milk (fresh)

Back to Risk-sharing

Risk-sharing regressions: Idiosyncratic Shocks

Shock	w (CFE)	$\log x$ (CRRA)
Death	0.023	0.020
	(0.050)	(0.036)
Death of earner	0.033	0.007
	(0.089)	(0.065)
Health	-0.056^{**}	-0.028
	(0.024)	(0.019)
Theft	0.023	0.022
	(0.033)	(0.026)
Income	0.056^{***}	0.075^{***}
	(0.007)	(0.006)

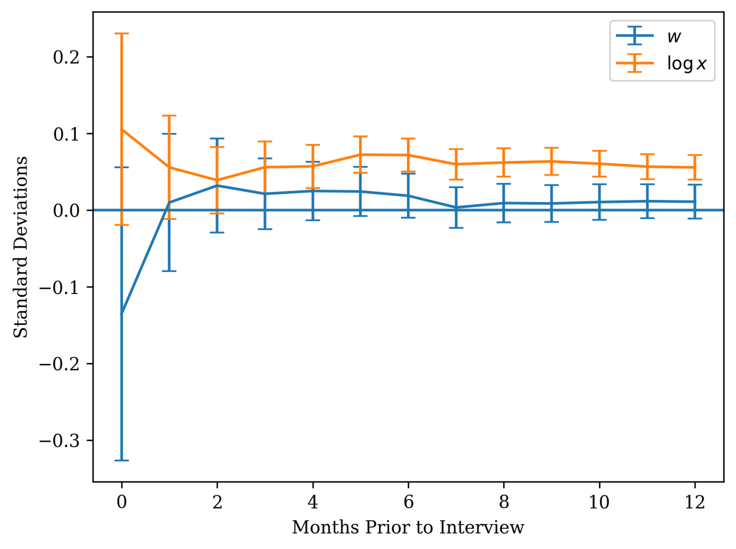
Risk-sharing regressions: Covariate Shocks

Shock	w (CFE)	$\log x$ (CRRA)
Drought	0.007	0.043^{***}
	(0.015)	(0.011)
Floods	0.030	0.091^{***}
	(0.033)	(0.027)
Pests	0.042	0.094^{***}
	(0.034)	(0.024)
Prices	-0.044	0.099^{***}
	(0.038)	(0.028)

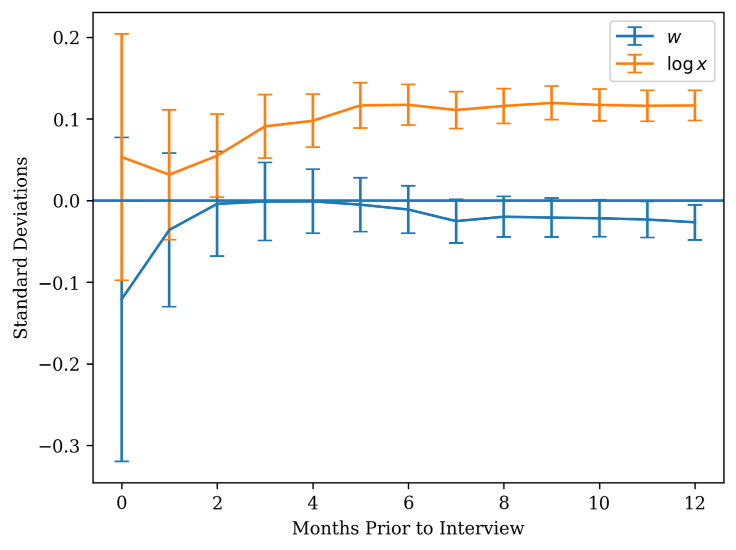
NB: Calculation of w already controls for prices!

Inference:
Households reporting covariate shocks affected principally via changes in prices, not idiosyncratic exposure to the shock.

Two-way FE: Covariate shocks and Idiosyncratic welfare



One-way FE: Covariate shocks and welfare



Conclusion

Takeaways

- MUEs (λ) appear in models all over the place.
- But conventional ways of measuring MUEs (e.g. CRRA) are very crude, and violate Engel's law.
- I find the most general class of preferences one can use to construct MUEs from expenditures.
- Preferences in this class take one of two forms, CFE or Generalized Stone-Geary.
- CFE lends itself to use in Townsend-style standard risk-sharing tests.
- In Uganda, covariate shocks' main effect is via prices. Policy implication is that interventions at market level may work as well as individual targeting.

Other ideas

Testing for "GE" effects
Egger et al. (2022) and Angelucci et al. (2018) measure or control for GE effects of shocks or large-scale experiments. This can be very expensive! This approach offers another way.

Derived Nutrient Demands
We have food conversion tables for East Africa that allow us to map food demands into "derived" nutrient demands, so we can see how nutrition responds to shocks.

Intra-household Pareto Weights
Since household composition affects demands for different foods differently, we can use this to identify within-household Pareto weights, which are central to models of intra-household allocation.

Other Dynamic Models
Use estimated MUEs as input into estimating more interesting dynamic models.

End Matter

References

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