Inside Out: The Allocative Impact of Firms' Make-or-Buy Decisions on Aggregate Energy Intensity

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Make-or-buy decisions allocate energy demand across heterogeneous firms

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Particularly relevant for energy in manufacturing: ▸ Energy intensity [Shapiro, 2021] ; ▸ Heterogeneity [Lyubich et al., 2018, De Lyon and Dechezleprêtre, 2025]

This project

Energy misallocation: Is the allocation of intermediate input production across heterogeneous firms *energy* efficient?

1.1 *Theoretical framework:* A network model of intermediate input trade and energy efficiency

1.2 *Quantitative analysis:* Using Italian firm-to-firm transaction data to infer in-house production and measure the implications for aggregate energy efficiency

Similar firms are extremely heterogeneous in energy intensity

Firms selling the same product

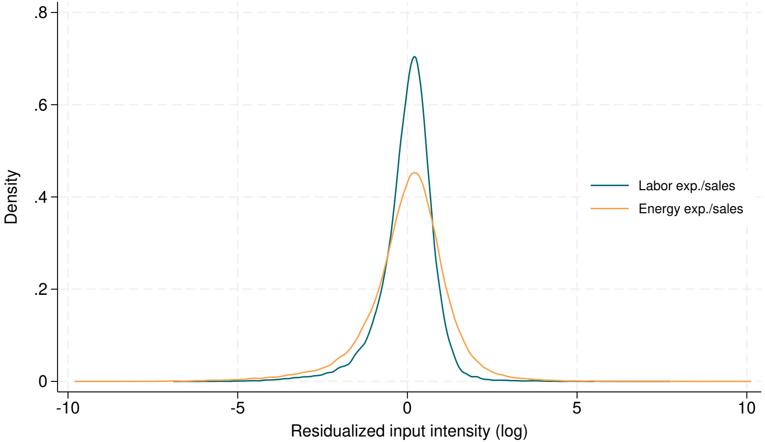

Figure 1: Distribution of Within-Product Input Intensity

Notes: The residuals are obtained from a cross-sectional (2019) regression of energy and labor expenditures to sales ratio on core product (6 digits) fixed effects, for the sample of single-product Italian manufacturing firms.

Potential producers are extremely heterogeneous in energy intensity

Firms producing the same input

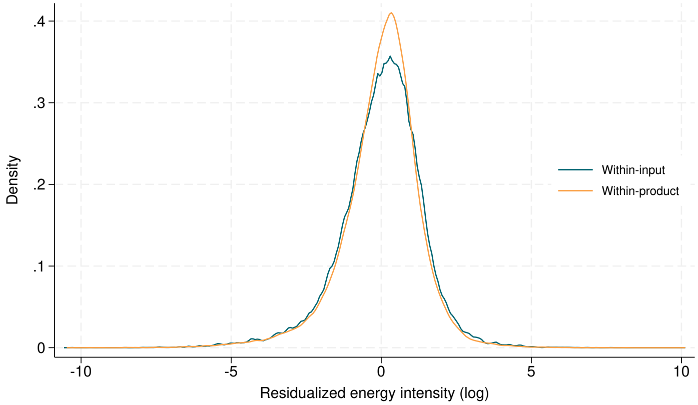

Figure 2: Dispersion in residualized energy intensity across sellers and across producers

Notes: The residuals are obtained from a cross-sectional (2019) regression of energy expenditures to sales ratio on input and core product (6 digits) fixed effects, the number of inputs produced in-house, and factor prices, for the sample of single-product Italian manufacturing firms. Within-product only considers firms for which the input is their core product. Labor

Literature

- **Misallocation:** [Restuccia and Rogerson, 2008, Hsieh and Klenow, 2009, Barrows and Ollivier, 2018, Schutze et al., 2019, Choi, 2020, Najjar and Cherniwchan, 2021, Caggese et al., 2023, Klenow et al., 2024, Kim, 2025] and **distortions in fragmented production:** [Liu, 2019, Baqaee and Farhi, 2020, Boehm and Oberfield, 2020]
  Contribution: Within-input misallocation as a source of energy misallocation
- **In-house production:** [Garg et al., 2019, Alfaro et al., 2019, Rachapalli, 2021, Boehm and Oberfield, 2020, Boehm and Oberfield, 2023]
  Contribution: Novel strategy to identify the *set* of inputs produced in-house
- **Make-or-buy decisions + energy outsourcing:**
  - Highly idiosyncratic [Antras et al., 2017, Blaum et al., 2018, Alfaro et al., 2019, Antràs, 2020, Boudreau et al., 2023]
  - Energy is one of many drivers of outsourcing: [Cole et al., 2014, Cherniwchan and Taylor, 2022, Dechezleprêtre et al., 2022, Choi et al., 2023, Levinson, 2023, Coster et al., 2024]

Today

Two-Stage Model

Intuitions about the Quantitative Strategy

Conclusion

Make or Buy? A simple static model of fragmentation

Downstream firm:

$$y = \begin{cases} \left(\beta^{1-\rho}(\nu_f e)^\rho + (1-\beta)^{1-\rho} l^\rho\right)^{\frac{1}{\rho}} & \text{if in-house} \\ q x_s & \text{if outsourced to } s \end{cases}$$

Where q represents relative outsourcing capability e.g., specialization gains ($q > 1$), and where $\rho < 0$ [Bretschger and Jo, 2021a, Känzig and Williamson, 2024] (WLOG). β is input-specific.

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Upstream firm: Firm s produces x_s and prices at marginal cost

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Make-or-Buy decision: The downstream firm produces in-house if $c^{\mathcal{I}} < \frac{c_s^{\mathcal{O}}}{q}$:

$$\underbrace{\left(\beta\left(\frac{p_e}{\nu_f}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta) w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}}_{\text{In-house}} < \underbrace{q^{-1}\left(\beta\left(\frac{p_e}{\nu_s}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta) w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}}_{\text{Outsource}}$$

$\Rightarrow$ In-house and outsourcing are perfect substitutes

Make-or-buy gains vs environmental externality

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- **Misallocation:** If $p_e < p_e^{SCC}$, the downstream firm gives an inefficiently large weight to q in its make-or-buy decision. Proof Fixed costs General Intuitions
  - Gains from $\nu_f > \nu_s$ are not large enough because energy is under-priced. Empirically: little consideration given to energy [Antonietti et al., 2017, Caron, 2022, Dussaux et al., 2023, Levinson, 2023]
  - If $p_e \to 0$ (emissions): Private decision based on q only

Make-or-buy gains vs environmental externality

Energy misallocation: The make-or-buy decision is inefficient if

$$\underbrace{\frac{1-\beta}{\beta}\left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}}}_{\text{Eq: outsourcing}} > \frac{\left(\left(q\frac{\nu_s}{\nu_f}\right)^{\frac{\rho}{1-\rho}}-1\right)\nu_f^{\frac{\rho}{1-\rho}}}{1-q^{\frac{\rho}{1-\rho}}} > \underbrace{\frac{1-\beta}{\beta}\left(\frac{w}{p_e^{SCC}}\right)^{\frac{-\rho}{1-\rho}}}_{\text{SP: in-house}}$$

1. **Heterogeneity:** If $\nu_f = \nu_s$, downstream firm's and social planner's decisions align $\Rightarrow$ efficient allocation *across* firms $\forall p_e$
2. **Environmental externality:** If $U(y,e) = U(y) \Rightarrow p_e = p_e^{SCC} \Rightarrow$ efficient allocation
   $\Rightarrow$ The equilibrium allocation maximizes output
3. **Firms distribution:** The joint distribution of $\left(\frac{\nu_f}{\nu_s}, q\right)$ determines the extent of between-firm misallocation

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Make-or-buy gains vs environmental externality

Suppose $\nu_s = 1$. For $p_e < p_e^{SCC}$:

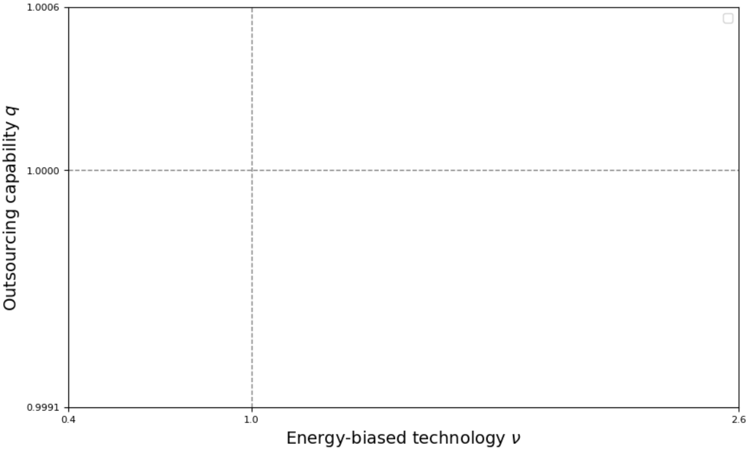

Figure 3: Specialization gains versus energy-biased technology

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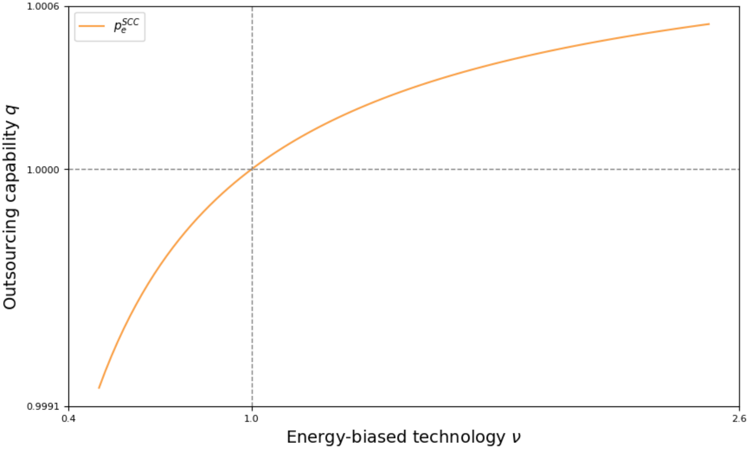

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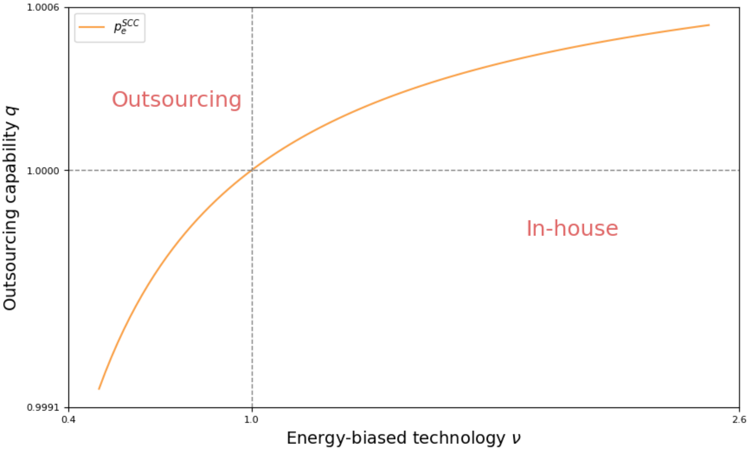

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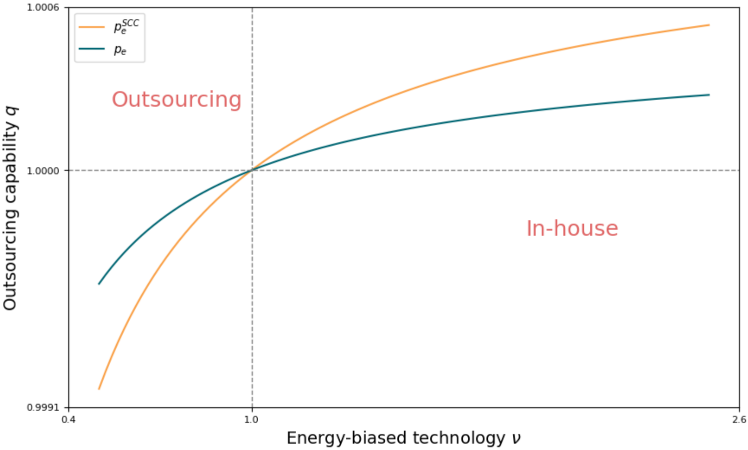

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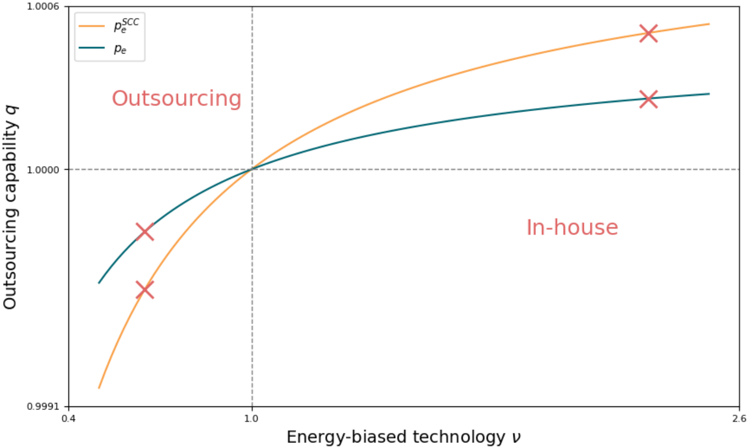

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$\downarrow p_e$: More weight to q relative to $\nu \Rightarrow$ need a lower relative q to compensate a higher relative ν

▸ Limits

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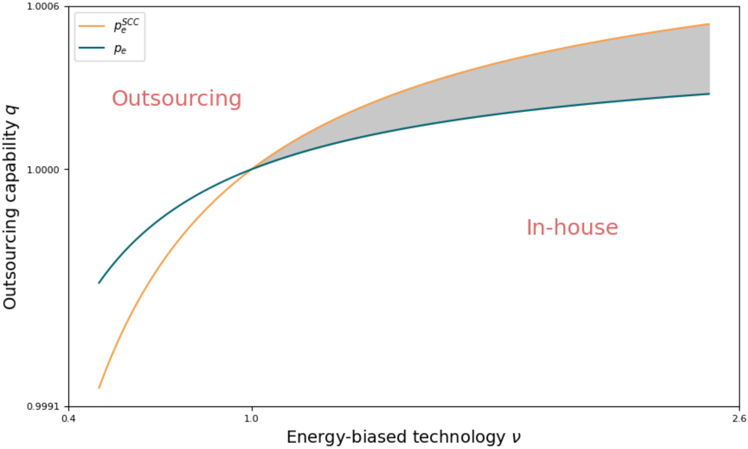

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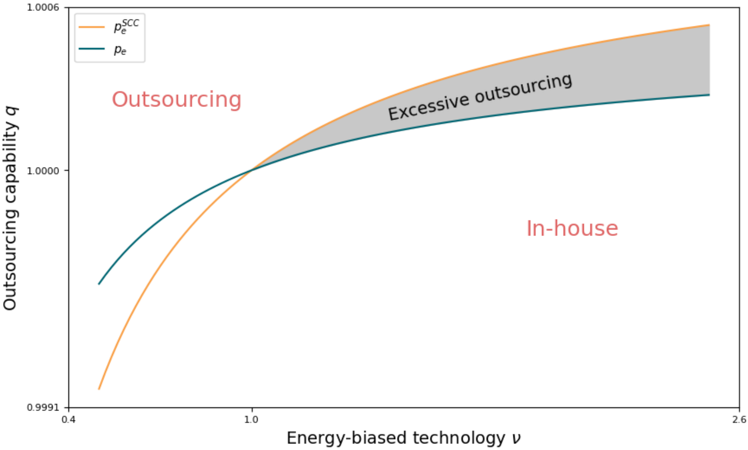

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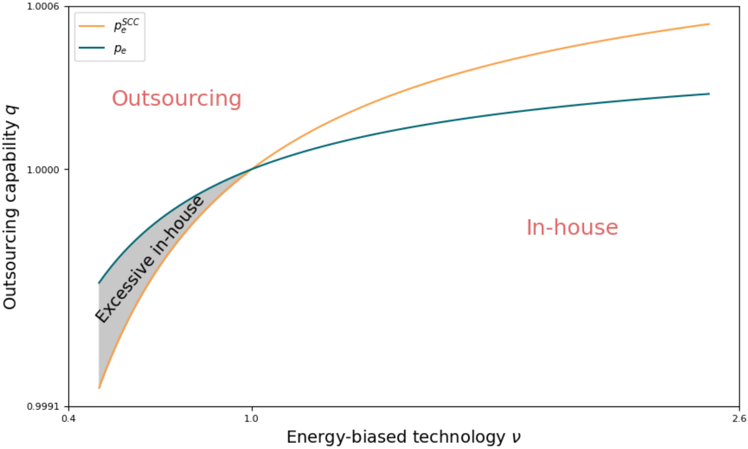

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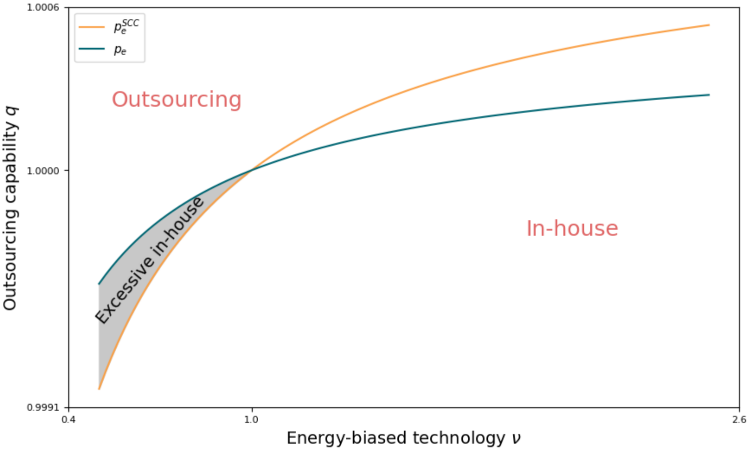

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⇒ The joint distribution of $\frac{\nu_f}{\nu_s}, q$ determines the extent of between-firm misallocation

Do energy efficient firms produce more in-house?

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- Energy intensive firms outsource less ...

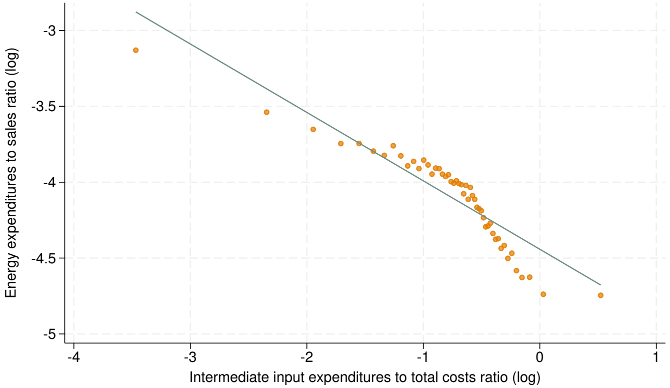


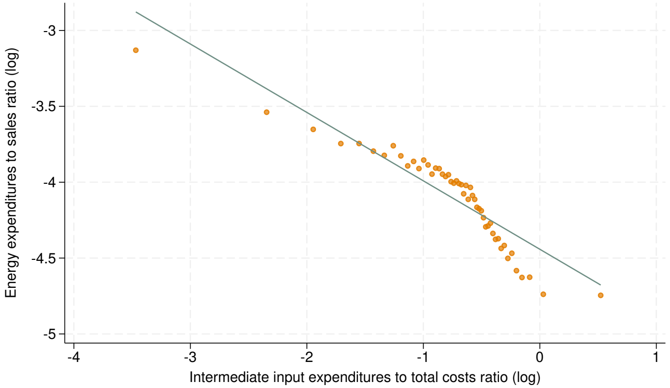
Figure 4: Correlation between firms' energy and intermediates expenditures share

Notes: The graph plots the relation between energy intensity, defined as energy expenditures per sales (in logarithm) and intermediate input expenditures to total costs (in logarithm). The relationship is measured controlling for sector (6-digit).

▸ Role ▸ VA ▸ RF ▸ Dispersion

Do energy efficient firms produce more in-house?

- Energy intensive firms outsource less ... but what about *effectively* energy efficient firms?



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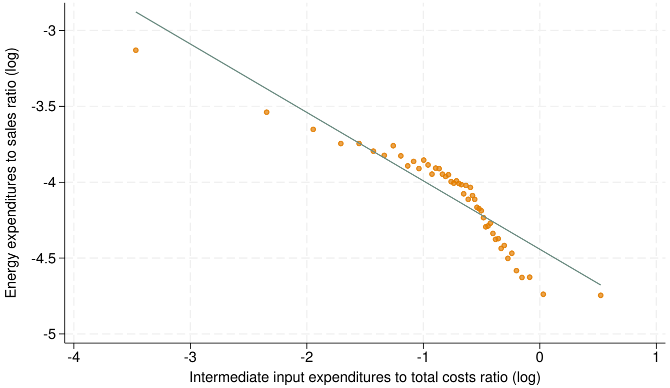
⇒ In-house production increases energy intensity regardless of the firm's effective energy efficiency



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Inefficient make-or-buy decision increases energy intensity

Suppose that the input is outsourced if $p_e < p_e^{SCC}$, but it is produced in-house under $p_e = p_e^{SCC}$.

$$\frac{(e/y)^*}{(e/y)^{SCC}} = \underbrace{\left(q_{ij}\frac{\nu_i}{\nu_j}\right)^{\frac{\rho}{1-\rho}} \times \left(\frac{c_i^{\mathcal{O}}(p_e^{SCC})/q_{ij}}{c_j^{\mathcal{I}}(p_e^{SCC})}\right)^{\frac{1}{1-\rho}}}_{\text{Extensive margin: Allocative efficiency}} \times \underbrace{\left(\frac{c_i^{\mathcal{O}}(p_e)/p_e}{c_i^{\mathcal{O}}(p_e^{SCC})/p_e^{SCC}}\right)^{\frac{1}{1-\rho}}}_{\text{Intensive margin}} > 1$$

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- **Intensive margin:** higher relative price of energy decreases energy demand: $\frac{c_i^{\mathcal{O}}(p_e)/p_e}{c_i^{\mathcal{O}}(p_e^{SCC})/p_e^{SCC}} > 1$

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- **Intensive margin:** higher relative price of energy decreases energy demand: $\frac{c_i^{\mathcal{O}}(p_e)/p_e}{c_i^{\mathcal{O}}(p_e^{SCC})/p_e^{SCC}} > 1$
- **Extensive margin:** make-or-buy decision reallocates production toward the more energy-efficient firm
  - **Direct effect of reallocation through technology:** Allocation choices imply $\left(q_{ij}\frac{\nu_i}{\nu_j}\right)^{\frac{\rho}{1-\rho}} > 1$; a more energy-efficient technology is used
  - **Effect of reallocation on costs:** $\frac{c_j^{\mathcal{O}}(p_e^{SCC})}{q_{ij}} > c_j^{\mathcal{I}}(p_e^{SCC}) > c_j^{\mathcal{I}}(p_e) > \frac{c_i^{\mathcal{O}}(p_e)}{q_{ij}} \Rightarrow$ a more productive firm produces

▸ Multiple inputs

Main Take-Away

- Lower energy intensity requires relatively energy efficient firms to produce, but distorted energy prices bias the make-or-buy decision away from $\frac{\nu_f}{\nu_s}$.
- The final extent of energy misallocation across firms depends on the joint distribution of $\left(q_{fs}, \frac{\nu_f}{\nu_s}\right)$.

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Mapping to the data

Italian data

1. **Firm-to-firm transaction data:** universe of the 2019 business-to-business (B2B) domestic transactions (with firm identifiers) ▸ Des. Stat.
   Expenditures on all inputs purchased *domestically* by all firms: $p_{ij}^{\mathcal{O}} x_{ij}^{\mathcal{O}}$ for $i \in \mathcal{O}_j$
2. **Custom data:** transactions with both extra-EU and intra-EU suppliers (with firm identifiers)
   Expenditures on all inputs purchased *abroad* by all firms: $p_{ij}^{O} x_{ij}^{O}$ for $i \in \mathcal{O}_j$
3. **Balance-sheet data:** universe of Italian limited liability companies
   Sales ($p_j y_j$), labor (l_j), wage (w_j), ...

Energy expenditures:

$$p_e e_j = \sum_{i \in \text{energy sec.}} p_{ij} x_{ij}$$

for all inputs $i \in$ energy sectors [Stangl et al., 2025]

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Exact calibration: each model firm corresponds to one empirical firm [Di Giovanni et al., 2024] ▸ Empirical units

Characterizing the allocation

Identifying firms' effective energy efficiency

1. $\nu_j \Rightarrow$ Firm-level residual variations in energy intensity

$$\Rightarrow log\frac{p_e e_j}{w l_j} = \frac{\rho}{1-\rho} log\left(\frac{w}{p_e}\right) + log\frac{\sum_{\omega\in\mathcal{I}_j}\beta_\omega}{\sum_{\omega\in\mathcal{I}_j} 1-\beta_\omega} + \underbrace{\frac{\rho}{1-\rho} log(\nu_j)}_{\text{residuals}}$$

2. $\mathcal{I}_j$: inferring in-house set $\mathcal{I}_j$ from product recipe Ω and inputs purchased $\mathcal{O}_j$

   ▸ details ▸ Recipes stat.

3. β_ω: Product fixed effects in $\frac{p_{e_{jt}} e_{jt}}{w_{jt} l_{jt}}$, for the subsample of firms with $\mathcal{I}_j = \{\omega_j\}$

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Identifying firms' in-house production

1. $\nu_j \Rightarrow$ Firm-level residual variations in energy intensity

$$\Rightarrow log\frac{p_e e_j}{wl_j} = \frac{\rho}{1-\rho}log\left(\frac{w}{p_e}\right) + \underbrace{log\frac{\sum_{\omega\in\mathcal{I}_j}\beta_\omega}{\sum_{\omega\in\mathcal{I}_j}1-\beta_\omega}}_{\text{In-house production}} + \underbrace{\frac{\rho}{1-\rho}log(\nu_j)}_{\text{residuals}}$$

2. $\mathcal{I}_j$: inferring in-house set $\mathcal{I}_j$ from product recipe Ω and inputs purchased $\mathcal{O}_j$

   ▸ details ▸ Recipes stat.

3. β_ω: Product fixed effects in $\frac{p_{e_{jt}}e_{jt}}{w_{jt}l_{jt}}$, for the subsample of firms with $\mathcal{I}_j = \{\omega_j\}$

   ▸ details

Characterizing the allocation

Identifying firms' in-house production

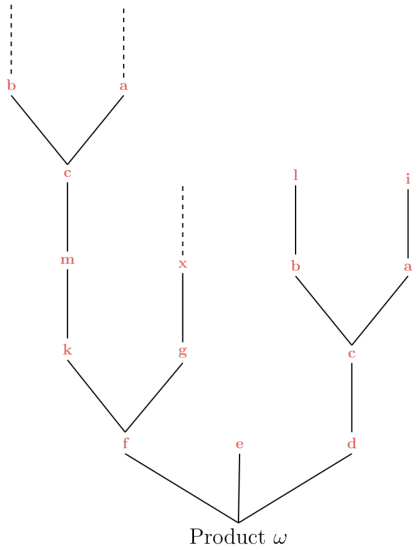


Figure 5: Production process of ω (Ω_ω)

▸ 6-digit ▸ Mp. firms ▸ Stats.

Characterizing the allocation

Identifying firms' in-house production

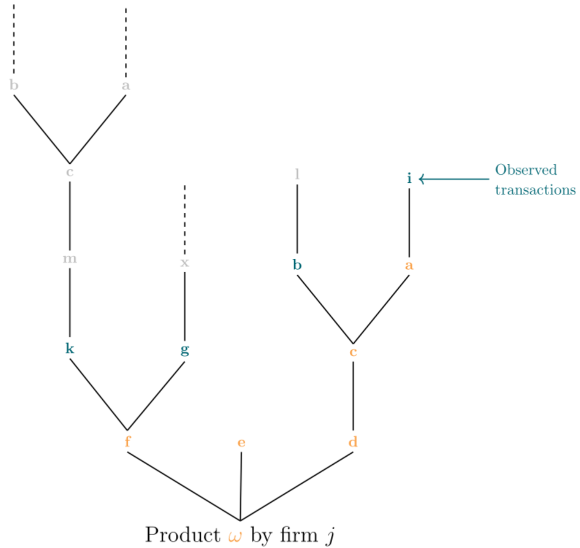


Figure 5: Production process of ω by firm j Des. Stats

Characterizing the allocation

Identifying firms' in-house production

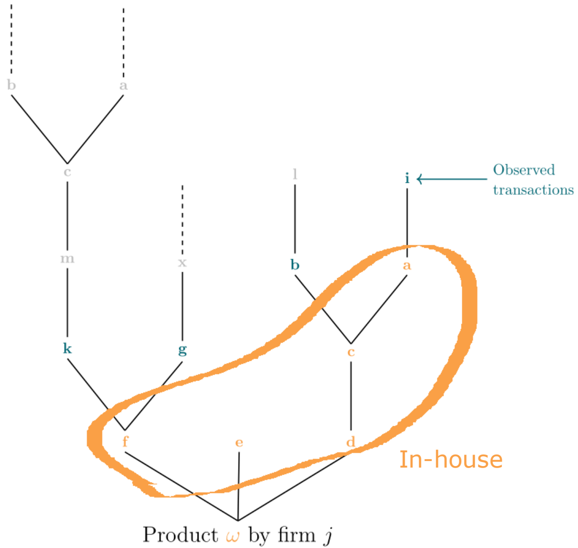


Figure 5: Production process of ω by firm j ▸ Des. Stats

Characterizing the allocation

Identifying firms' in-house production

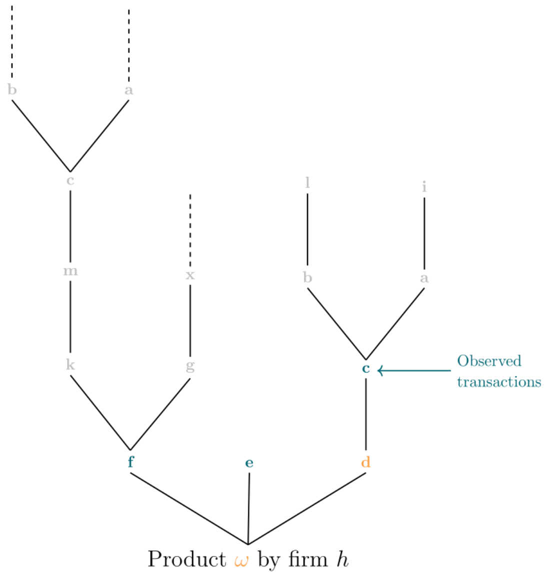


Figure 5: Production process of ω by firm h ▸ Des. Stats

Characterizing the allocation

Identifying firms' in-house production

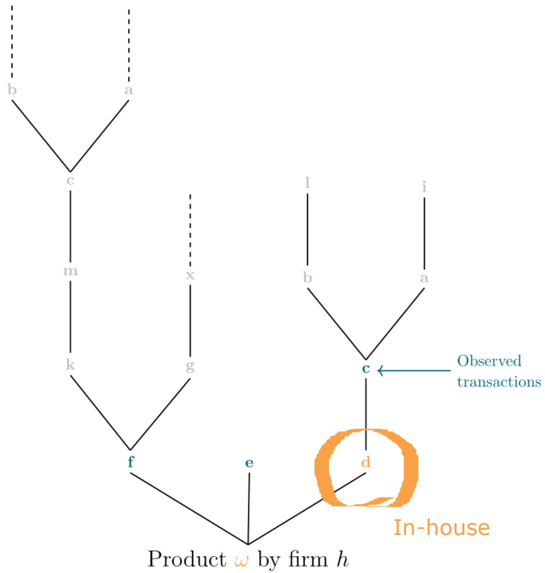


Figure 5: Production process of ω by firm h ▸ Des. Stats

Characterizing the allocation

Identifying firms' in-house production

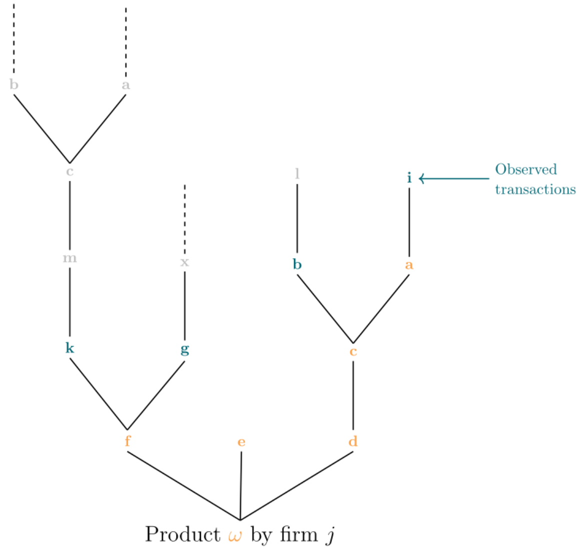
Recursive iterations: Move upstream and consider direct links only:

1. Start from core product ω and consider all direct inputs of product ω ($\hat{\omega} \in \mathcal{U}_{\omega}$): $\hat{\omega} \in \mathcal{I}_j$ if $\hat{\omega} \notin \mathcal{O}_j$ and $\hat{\omega} \in \mathcal{D}_{\tilde{\omega}}$ $\forall \tilde{\omega} \in \mathcal{O}_j$, or $\hat{\omega} \in \mathcal{R}$
2. Move upstream: in-house inputs $\hat{\omega} \in \mathcal{I}_j$ become the "products", and repeat step 1 for each $\hat{\omega} \in \mathcal{I}_j$.
3. Move upstream until $\mathcal{I}_j$ is left unchanged.

Details

Characterizing the allocation

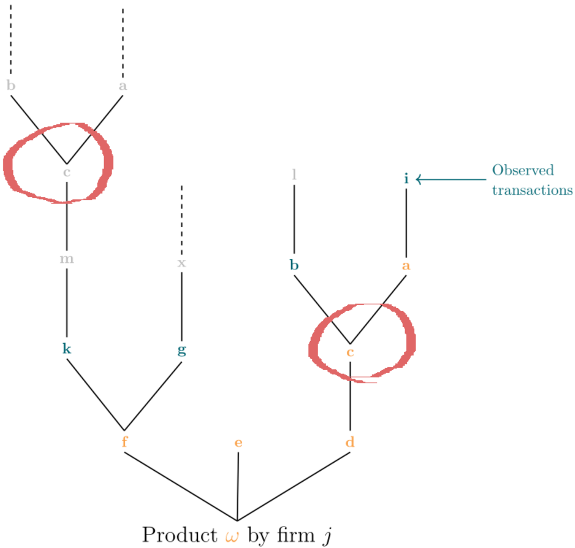
Identifying firms' in-house production



Characterizing the allocation

Identifying firms' in-house production

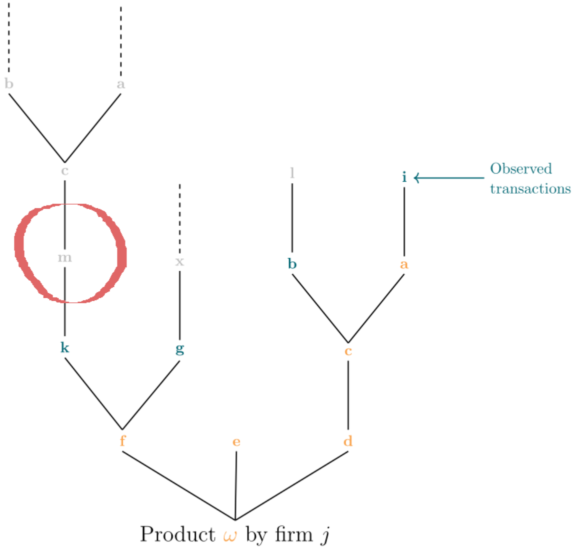
If $\mathcal{I}_j = \Omega_\omega \backslash \{ \cup_{\hat{\omega} \in \mathcal{O}_j} (\hat{\omega} \cup \Omega_{\hat{\omega}}) \}$:



Characterizing the allocation

Identifying firms' in-house production

If $\mathcal{I}_j = \left\{ \cup_{\hat{\omega} \in \mathcal{O}_j \cap \Omega_\omega} \mathcal{D}_{\hat{\omega}} \setminus \{\omega \cup \mathcal{D}_\omega\} \right\}$:



Characterizing the allocation

Identifying firms' in-house production

Firm	Core product	Product recipe	Outsourced inputs	#In-house	In-house inputs
A	Fire detector	{ rubber, plastic sheets, coke, jacks, dryers, electrical fuses, electrical circuits, nails, cutting blades, alarm device, office machinery, fiber optic cable, battery chargers, interchangeable tools, hinges, smoke measurement, sound instrument, hydraulic valve, . . . }	{ smoke measurement, hinges, battery chargers, hydraulic valve, . . . }	1	{ alarm device }
B	Fire detector	{ rubber, plastic sheets, coke, jacks, dryers, electrical fuses, electrical circuits, nails, cutting blades, alarm device, office machinery, fiber optic cable, battery chargers, interchangeable tools, hinges, smoke measurement, sound instrument, hydraulic valve, . . . }	{ alarm device, battery chargers, electrical fuses, electrical circuits, plastic sheets, . . . }	11	{ fiber optic cable, cutting blades, nails, hinges, interchangeable tools, sound instrument, hydraulic valve, . . . }

Characterizing the allocation

Identifying firms' in-house production

1. $\nu_j \Rightarrow$ Firm-level residual variations in energy intensity

$$\Rightarrow log\frac{p_e e_j}{wl_j} = \frac{\rho}{1-\rho}log\left(\frac{w}{p_e}\right) + \underbrace{log\frac{\sum_{\omega\in\mathcal{I}_j}\beta_\omega}{\sum_{\omega\in\mathcal{I}_j}1-\beta_\omega}}_{\text{In-house production}} + \underbrace{\frac{\rho}{1-\rho}log(\nu_j)}_{\text{residuals}}$$

2. $\mathcal{I}_j$: inferring in-house set $\mathcal{I}_j$ from product recipe Ω and inputs purchased $\mathcal{O}_j$
   ▸ details ▸ Recipes stat.

3. β_ω: Product fixed effects in $\frac{p_{e_{jt}}e_{jt}}{w_{jt}l_{jt}}$, for the subsample of firms with $\mathcal{I}_j = \{\omega_j\}$
   ▸ details

Energy-biased technology and in-house production

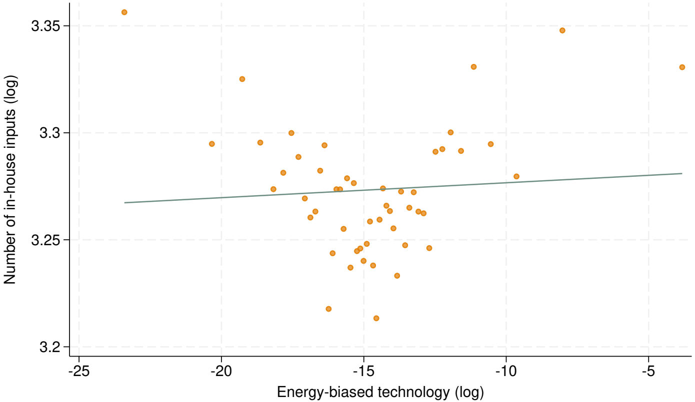


Figure 5: Correlation between firms' energy-biased technology and number of in-house inputs

Notes: The graph plots the relation between firms' energy-biased technology and the number of inputs produced in-house, both computed by the authors. The relation controls for region and product fixed effects. ▸ product vs input

Quantifying misallocation

Inferring the outsourcing capability

Counterfactual allocation requires characterizing *all* potential relationships

- q_{ij}: buyer×supplier-level residuals

Quantifying misallocation

Inferring the outsourcing capability

Counterfactual allocation requires characterizing *all* potential relationships

- q_{ij}: buyer×supplier-level residuals
  - ⇒ Need to infer the unobserved q_{ij}:

Quantifying misallocation

Inferring the outsourcing capability

Counterfactual allocation requires characterizing *all* potential relationships

- q_{ij}: buyer×supplier-level residuals
  - ⇒ Need to infer the unobserved q_{ij}:
    1. Selection on observables: $log(q_{ij}) = z_i + z_j + \mathbf{X}_{ij}\delta + \epsilon_{ij}$

Quantifying misallocation

Inferring the outsourcing capability

Counterfactual allocation requires characterizing *all* potential relationships

- q_{ij}: buyer×supplier-level residuals

  $\Rightarrow$ Need to infer the unobserved q_{ij}:

  1. Selection on observables: $log(q_{ij}) = z_i + z_j + \mathbf{X}_{ij}\delta + \epsilon_{ij}$
  2. Revealed preference: Suppose that $\frac{1}{q_{ij}} \sim Pareto(z_m, \theta)$:

▸ Details

Quantifying misallocation

Inferring the outsourcing capability

Counterfactual allocation requires characterizing *all* potential relationships

- q_{ij}: buyer×supplier-level residuals
  - ⇒ Need to infer the unobserved q_{ij}:
    1. Selection on observables: $log(q_{ij}) = z_i + z_j + \mathbf{X}_{ij}\delta + \epsilon_{ij}$
    2. Revealed preference: Suppose that $\frac{1}{q_{ij}} \sim Pareto(z_m, \theta)$: Details

    2.1 **Outsourced inputs:** For each potential supplier s that was not chosen against supplier i, the conditional distribution is $\frac{1}{q_{sj}} \sim Pareto\left(\frac{c_i(\omega)/q_{ij}}{c_s(\omega)}, \theta\right)$

    2.2 **In-house inputs:** For each potential supplier s, the conditional distribution is $\frac{1}{q_{ij}} \sim Pareto\left(\frac{c_j^{\mathcal{I}}(\omega)}{c_i(\omega)}, \theta\right)$

Quantifying misallocation

Inferring the outsourcing capability

Counterfactual allocation requires characterizing *all* potential relationships

- q_{ij}: buyer×supplier-level residuals

⇒ Need to infer the unobserved q_{ij}:

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2.1 **Outsourced inputs:** For each potential supplier s that was not chosen against supplier i, the conditional distribution is $\frac{1}{q_{sj}} \sim Pareto\left(\frac{c_i(\omega)/q_{ij}}{c_s(\omega)}, \theta\right)$

2.2 **In-house inputs:** For each potential supplier s, the conditional distribution is $\frac{1}{q_{ij}} \sim Pareto\left(\frac{c_j^{\mathcal{I}}(\omega)}{c_i(\omega)}, \theta\right)$

⇒ Draw a q_{ij} from those conditional distributions for all firms and inputs.

⇒ All model simulations are data-consistent

▸ Full Model

Today

Two-Stage Model

Intuitions about the Quantitative Strategy

Conclusion

Conclusion

- Large heterogeneity in firm-level energy-biased technology means that the allocation of energy matters for aggregate outcomes
- In-house intermediate input production is an important determinant of firm-level energy demand
  $\Rightarrow$ Make-or-buy decisions allocate energy across firms
- ⚠ Distorted energy prices lead to under-consideration of the energy component in the make-or-buy decision $\Rightarrow$ between-firm, within-input, energy misallocation
- **Next steps:**
  1. Calibration of the remaining relevant firm-level parameters using Italian B2B data
  2. Quantify aggregate energy intensity and welfare loss relative to a counterfactual efficient allocation

Conclusion

- Large heterogeneity in firm-level energy-biased technology means that the allocation of energy matters for aggregate outcomes
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- **Next steps:**
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  2. Quantify aggregate energy intensity and welfare loss relative to a counterfactual efficient allocation

Thank you very much!

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Today

Motivations

- Descriptive Statistics
- In-house production

Two-Stage Model

Multiple-Stage Model

Details on the Multiple-stage model

Calibration

Role of the make-or-buy decision

1. **Measurement:** Quantifying misallocation requires knowing the effective energy efficiency of each potential producer:
    - $\frac{e_j}{y_j} = F(\mathcal{I}_j, \nu_j)$: need the make-or-buy outcome ($\mathcal{I}_j$) to recover the effective energy-biased technology ν_j
2. **Within-input heterogeneity:** Potential supplier comparison requires knowing *which* input is produced in-house

$\Rightarrow$ Novel method to infer the *set* $\mathcal{I}_j$ of in-house inputs from recipes and observed transactions

(Not today) Investment incentives:

1. In-house = investments in energy-biased technology spread across more inputs
2. In-house = no sales $\Rightarrow$ less production of each input

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Potential producers are extremely heterogeneous in energy intensity

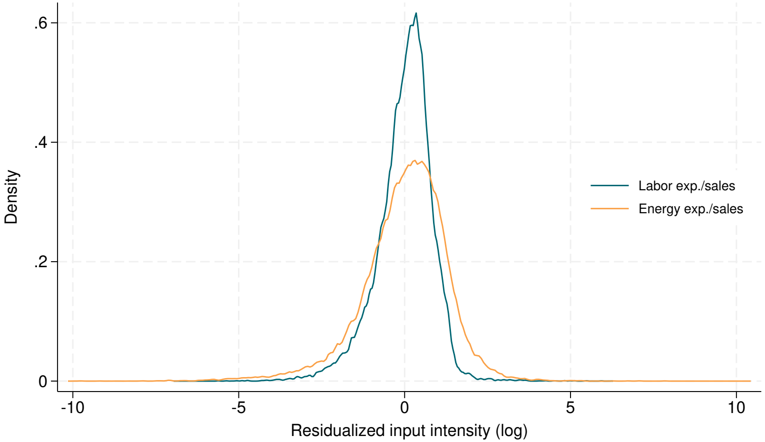

Figure 6: Distribution of Within-Input Input Intensity

Notes: The residuals are obtained from a cross-sectional (2019) regression of $\frac{p_E E}{sales}$ and $\frac{wL}{sales}$ on product (6 digits).

▸ Back

Intermediate input production is energy intensive

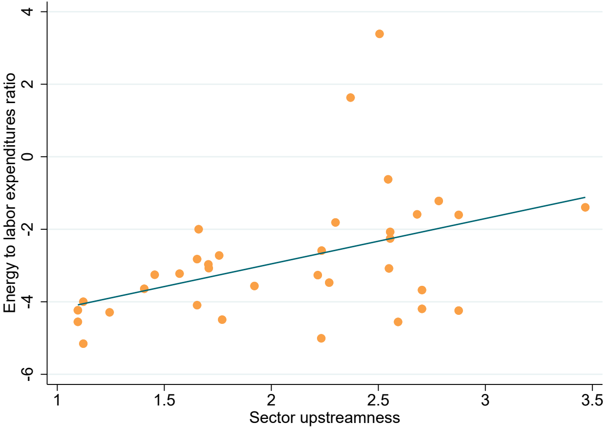

Figure 7: Correlation between the sectoral energy to labor expenditure ratio and the sector's own upstreamness

Notes: Source: Eurostat Italian SUT (2019). The graph plots the relation between the average sectoral energy expenditures to labor expenditures ratio (in log) and the sectoral upstreamness index of [Antràs et al., 2012]. Back

Intermediate input production is energy intensive

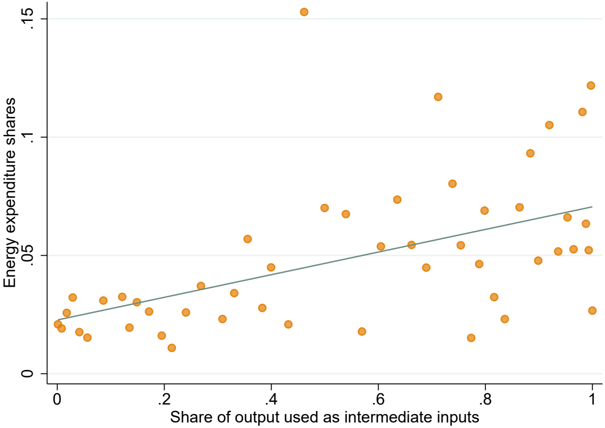

Figure 8: Correlation between the sectoral energy expenditure share and the sector's intermediates output share

Notes: Source: US BEA IO tables (2018). The graph plots the relation between the average sectoral energy to total expenditures ratio and the sectoral share of output used as intermediates by other industries. ▸ Back

Similar firms are heterogeneous in their make-or-buy decisions I

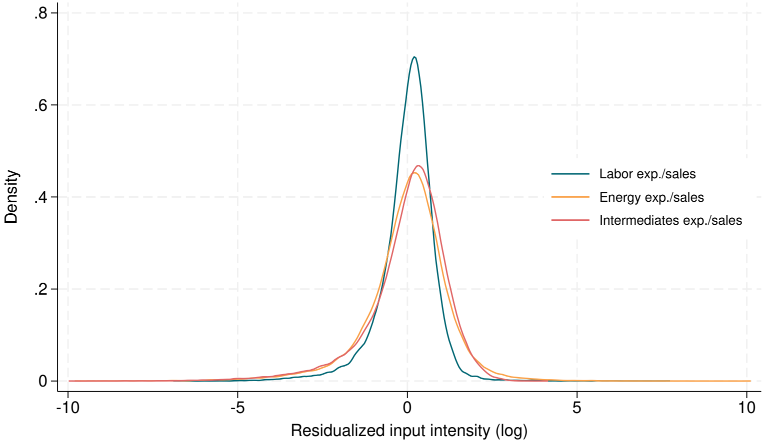

Figure 9: Distribution of residualized input intensity

Notes: The residuals are obtained from a cross-sectional (2019) regression of the input expenditures to sales ratio on product (6 digits) fixed effects.

▸ Back

Similar firms are heterogeneous in their make-or-buy decisions II

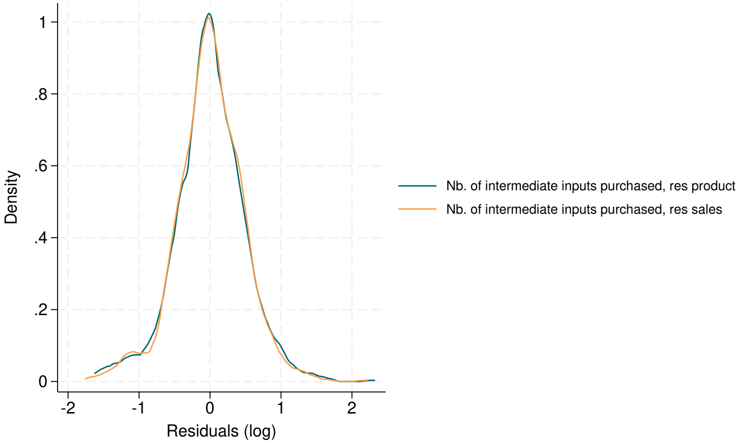

Figure 10: Distribution of residualized number of inputs purchased

Notes: The residuals are obtained from a cross-sectional (2019) regression of the number of distinct inputs purchased by the firms, on product (6 digits) and sales. ▸ Back

Make-or-buy decisions allocate energy across firms I

Outsourcing is a substitute to energy

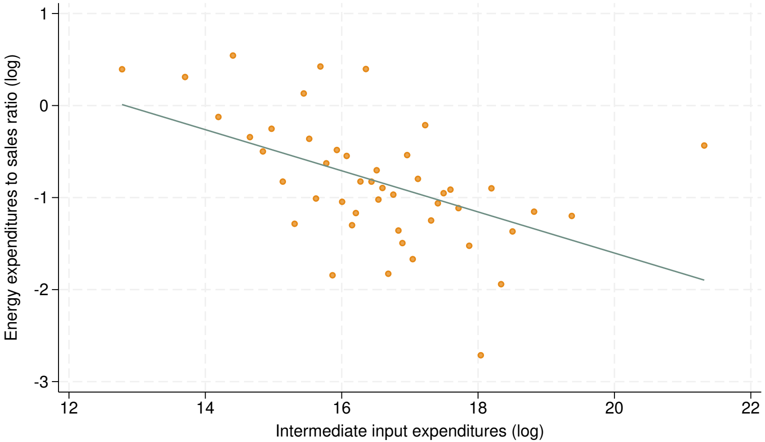

Figure 11: Correlation between firms' energy per unit sold and intermediate input expenditures

Notes: The graph plots the relation between energy intensity, defined as energy expenditures per sales (in logarithm) and intermediate input expenditures (in logarithm). The relationship is measured controlling for product (6-digit). ▸ Back

Make-or-buy decisions allocate energy across firms II

Outsourcing is a substitute to energy

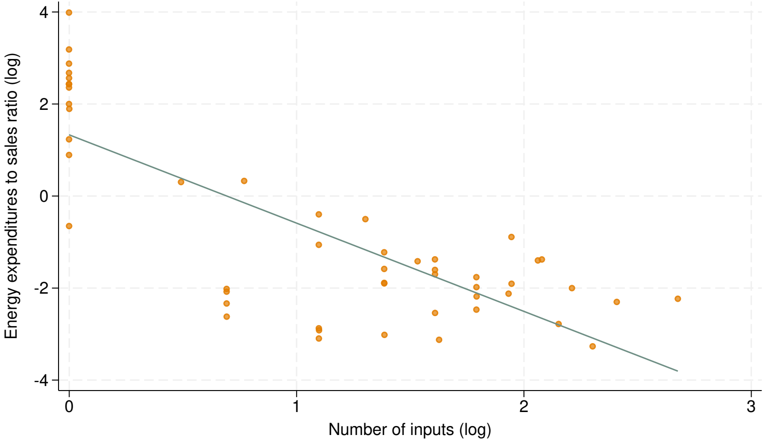

Figure 12: Correlation between firms' energy per unit sold and the number of intermediate inputs purchased

Notes: The graph plots the relation between energy intensity, defined as energy expenditures per sales (in logarithm) and the number of inputs purchased (in logarithm). The relationship is measured controlling for product (6-digit) and sales. Back

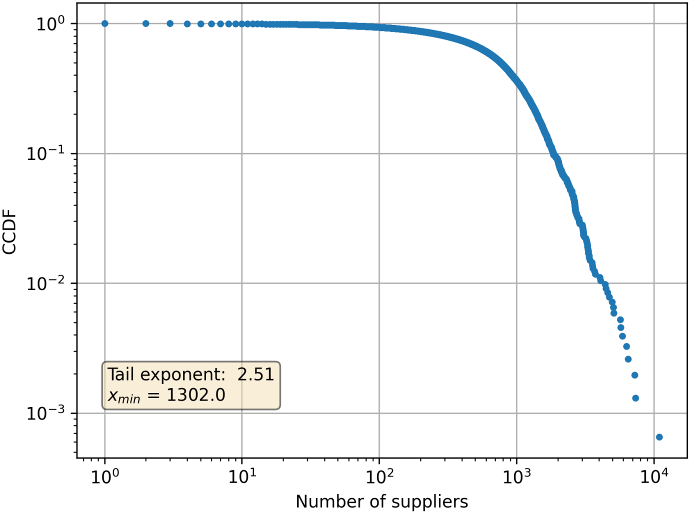

Figure 13: Distribution of the number of suppliers per firm

Notes: This figure plots the CCDF of the number of distinct suppliers (for any type of inputs) per firm. Source: Italian VAT data. Computations by the authors.

Descriptive Statistics on the Italian B2B Data II

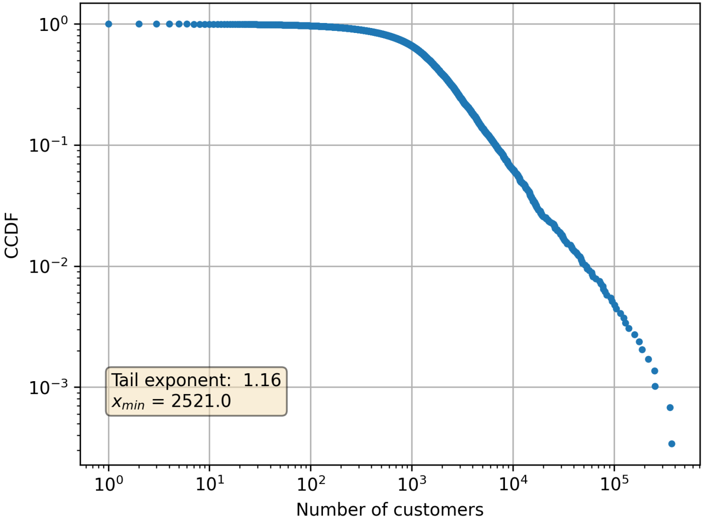

Figure 14: Distribution of the number of buyers per firm

Notes: This figure plots the CCDF of the number of distinct buyers per firm. Source: Italian VAT data. Computations by the authors.

Descriptive Statistics on the Italian B2B Data III

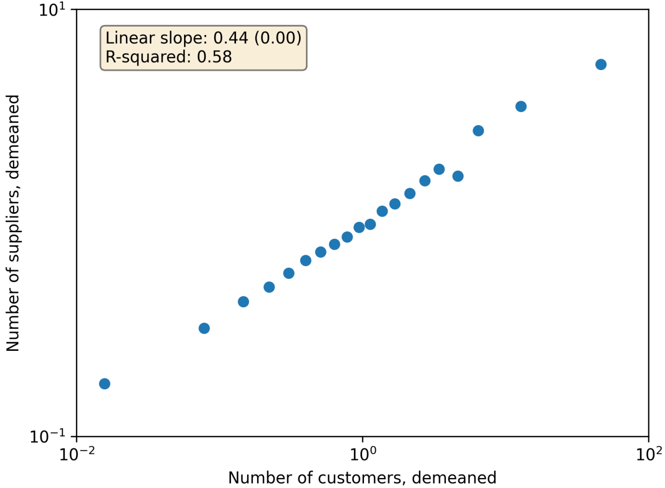

Figure 15: Correlation between the demeaned number of buyers and suppliers per firm

Notes: This figure plots the number of distinct suppliers against the number of distinct buyers per firm. Both variables are net of the industry average.

Source: Italian VAT data. Computations by the authors. ▸ Back

Descriptive Statistics on the Italian B2B Data IV

Table 1: Network statistics: suppliers and buyers

Statistic	Mean	Std. dev.	Min	P25	P50	P75	Max
Number of buyers per domestic seller	19.56	49.57	1	2	6	19	4916
Number of suppliers per domestic buyer	16.57	21.96	1	4	10	21	852
Number of suppliers per buyer $\times$ input	2.47	3.48	1	1	1	3	336
Number of domestic suppliers per buyer $\times$ input	2.47	3.50	1	1	1	3	336

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Energy Efficiency is a Firm-level Characteristic

Example

German steelmaker Salzgitter will start converting its steel production facilities to run on green hydrogen-based technology by the end of 2025, after it received confirmation that it will get almost €1bn ($1.1bn) in government subsidies.

"SALCOS® is being realized in stages and it consists of a direct reduction plant, an electric arc furnace (both already under construction) and the 100 MW electrolysis plant for the production of hydrogen. Conversion to virtually carbon-free steel production at the Salzgitter facility is due to be completed by 2033."

Back

In-house production is an important determinant of firms' energy demand

Example

Input i requires labor and energy, with α_i increasing with upstreamness:

$$x_i = l_i^{1-\alpha_i-\beta} e_i^{\alpha_i} x_{i-1}^{\beta}$$

For firm j producing good i:

$$\frac{p_e e_j^h}{p x_j} = \alpha_i + \alpha_{i-1}\beta \quad \text{if } i-1 \text{ in-house}$$

$$\frac{p_e e_j^o}{p x_j} = \alpha_i \quad \text{if } i-1 \text{ outsourced}$$

$$\frac{w l_j^h}{p x_j} = (1-\beta)(1+\beta) - \alpha_i - \alpha_{i-1}\beta \quad \text{if } i-1 \text{ in-house}$$

$$\frac{w l_j^o}{p x_j} = 1 - \alpha_i - \beta \quad \text{if } i-1 \text{ outsourced}$$

Therefore, for a given level of production x_j

$$\frac{e_j^h - e_j^o}{e_j^o} = \frac{\alpha_{i-1}\beta}{\alpha_i} > \frac{l_j^h - l_j^o}{l_j^o} = \frac{(1-\alpha_{i-1}-\beta)\beta}{1-\alpha_i-\beta}$$

Since $\alpha_{i-1} > \alpha_i$. $\Rightarrow$ Whether firm j produces $i-1$ in-house or outsources it is relevant predominantly for its energy demand.

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Today

Welfare: Representative household utility

$$U\left(\underset{(+)}{y}, \underset{(-)}{e}\right) \Rightarrow p_e^{SCC} = -\frac{\partial U/\partial e}{\partial U/\partial y}$$

- ⇒ Energy externality creates a distortion
- ⇒ Welfare gains ≠ output gains (output is maximized) as in spatial equilibrium model [Donald et al., 2025]

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Today

Make-or-buy gains vs environmental externality

Intuitions: Between-firm allocation minimizes

$$\text{Energy intensity:} \quad \min_{i=\{s,f\}} (q_{if}\nu_i)^{\frac{\rho}{1-\rho}} \frac{\beta}{1-\beta}\left(\frac{p_e}{w}\right)^{\frac{-\rho}{1-\rho}}$$

$$\text{Unit cost:} \quad \min_{i=\{s,f\}} (q_{if}\nu_i)^{\frac{\rho}{1-\rho}} \frac{\beta}{1-\beta}\left(\frac{p_e}{w}\right)^{\frac{-\rho}{1-\rho}} + \underbrace{q_{if}^{\frac{\rho}{1-\rho}}}_{Wedge}$$

1. **Energy wedge:** Decisions differ due to the outsourcing capabilities q_{ij}
2. **Factor weights:** The larger the energy price p_e, the more aligned the two decisions
   $\Rightarrow$ At low energy prices, the relative gains from $\nu_f > \nu_s$ are small compared to gains from $q_{fs} > q_{ff} = 1$
3. **Joint distribution:** Decisions align if $cov(\nu_s, q_{sf}) > 0$

Back

Outsourcing decision: Firm j produces input ω in-house if $\frac{c_j^{\mathcal{I}}(\omega)}{q_{jj}} < \frac{c_i^{\mathcal{O}}(\omega)}{q_{ij}}$:

$$q_{jj}^{-1}\left(\beta_\omega\left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}} < q_{ij}^{-1}\left(\beta_\omega\left(\frac{p_e}{\nu_i}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho}{\rho}}$$

1. If $p_e = p_e^{SCC}$: no distortion.
   1.1 **The equilibrium allocation is efficient**, given the network (First Welfare Theorem). *Proof.* The social planner maximizes an objective function $W(y,e)$ where $\frac{\partial W}{\partial e} < 0$. subject to $y_j = q_{kj}\left(\beta_\omega^{1-\rho}(\nu_k e_{ij})^\rho + (1-\beta_\omega)^{1-\rho} l_{ij}^\rho\right)^{\frac{1}{\rho}}$ for $k = \{i, j\}$ and $l_j = \bar{L}$. For the decentralized equilibrium to match the social planner allocation, $p_e = \frac{\partial W/\partial e}{\partial W/\partial y} = SCC$ and $w = \frac{\mu_l}{\partial W/\partial y}$ where μ_l is the Lagrange multiplier associated to the labor endowment constraint.

1.2 **The outsourcing decision is efficient.**
Proof. Since energy decreases welfare, the social planner wants to minimize energy used to produce one unit of output when choosing which firm should produce. The efficient allocation of energy and labor yields the following resources needed to produce one unit of output:

$$1 \leq q_{kj}^{-1}\left(\beta_\omega\left(\frac{\frac{\partial W/\partial e}{\partial W/\partial y}}{\nu_k}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega)\left(\frac{\mu_l}{\partial W/\partial y}\right)^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}$$

Which is minimized by choosing the firm $k = \{i, j\}$ with the lowest unit cost of production c_{ij}, that is to solve for $\min\left\{\frac{c_j^{\mathcal{I}}(\omega)}{q_{jj}}, \frac{c_i^{\mathcal{O}}(\omega)}{q_{ij}}\right\}$.

2. If $p_e < p_e^{SCC}$:

   2.1 **Insufficient weight is given to ν in the outsourcing decision when $\rho < 0$.**
   Proof. The make-or-buy decision can be rewritten as

$$\left(\left(\frac{q_{ij}}{q_{jj}}\right)^{\frac{-\rho}{1-\rho}} \nu_j^{\frac{\rho}{1-\rho}} - \nu_i^{\frac{\rho}{1-\rho}}\right) \beta_\omega p_e^{\frac{-\rho}{1-\rho}} + \left(\left(\frac{q_{ij}}{q_{jj}}\right)^{\frac{-\rho}{1-\rho}} - 1\right)(1-\beta_\omega) w^{\frac{\cdot\rho}{1-\rho}} < 0$$

   Normalize $\nu_j = 1$. We can derive $\frac{dLHS}{dlog\nu_i/\nu_j} = -\frac{\rho}{1-\rho}\beta_\omega \left(\frac{p_e}{\nu_i}\right)^{\frac{-\rho}{1-\rho}}$ and $\frac{dLHS}{dlogq_{ij}/q_{jj}} = \frac{-\rho}{1-\rho}\left(\frac{q_{ij}}{q_{jj}}\right)^{\frac{-\rho}{1-\rho}} \left(\beta_\omega p_e^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega) w^{\frac{-\rho}{1-\rho}}\right)$. Thus,

$$\frac{\frac{dLHS}{dlogq_{ij}/q_{jj}}}{\frac{dLHS}{dlog\nu_i/\nu_j}} = \left(\frac{q_{ij}}{q_{jj}\nu_i}\right)^{\frac{-\rho}{1-\rho}} \left(1 + \frac{1-\beta_\omega}{\beta_\omega}\left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}}\right)$$

   Which increases with p_e if $\rho < 0$. The decision is more sensitive to the capabilities ratio $\frac{q_{ij}}{q_{jj}}$ (relative to its sensitivity to the energy-biased technology ratio $\frac{\nu_j}{\nu_i}$) the lower p_e is.

3. **Relatively more weight is given to ν in the outsourcing decision of high β_ω input.** $\frac{\frac{dLHS}{dlogq_{ij}/q_{jj}}}{\frac{dLHS}{dlog\nu_j/\nu_i}}$ is smaller in magnitude at large β_ω.

Back

Outsourcing decision: Firm j produces input ω in-house if $\frac{c_j^{\mathcal{I}}(\omega)}{q_{jj}} < \frac{c_i^{\mathcal{O}}(\omega)}{q_{ij}}$:

$$q_{jj}^{-1}\left(\beta_\omega\left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}} < q_{ij}^{-1}\left(\beta_\omega\left(\frac{p_e}{\nu_i}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}$$

$\Rightarrow$ The Social Planner would have to tax firm-to-firm transaction (i.e., $\tilde{q}_{ij} = (1-\tau_{ij})q_{ij}$) by

$$\tau_{ij} = 1 - \frac{q_{ij}}{q_{jj}} \frac{\left(\beta_\omega\left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}}{\left(\beta_\omega\left(\frac{p_e}{\nu_i}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}}$$

To induce in-house production (= the efficient allocation).

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Specialization gains vs environmental externality I

Leontieff Example

Downstream firm: produces using an intermediate input ω: $y_j = x_j(\omega)$

Intermediate input: can be produced in-house by firm j or outsourced to specialized upstream firm i that prices at marginal cost.

$$x_{kj}(\omega) = q_{kj} \min(\nu_k e_{kj}(\omega), l_{kj}(\omega)) \quad \text{for } k = \{i, j\}$$

Where $q_{ij} > q_{jj} = 1$ represents specialization gains.

Fragmentation decision: Firm j produces input i in-house if $\frac{c_j^{\mathcal{I}}(\omega)}{q_{jj}} < \frac{c_i^{\mathcal{O}}(\omega)}{q_{ij}}$:

$$\frac{1}{q_{jj}}\left(\frac{p_e}{\nu_j} + w\right) < \frac{1}{q_{ij}}\left(\frac{p_e}{\nu_i} + w\right)$$

Specialization gains vs environmental externality II

Leontieff Example

If $p_e < p_e^{SCC}$, insufficient consideration is given to the energy component in the fragmentation decision:

$$\frac{\partial \frac{c}{q} / \partial log q}{\partial \frac{c}{q} / \partial log \nu} = 1 + \frac{w\nu}{p_e}$$

Which decreases in p_e.

The equilibrium fragmentation decision is inefficient if

$$\underbrace{\frac{p_e}{w} <}_{\text{Eq: outsourcing}} \quad \frac{\frac{q_{ij}}{q_{jj}} - 1}{1 - \frac{q_{ij}\nu_i}{q_{jj}\nu_j}} \underbrace{< \frac{p_e^{SCC}}{w}}_{\text{SP: in-house}}$$

Which can only arise if $\frac{\nu_j}{\nu_i} > (<) \frac{q_{ij}}{q_{jj}} > (<) 1$ If $\nu_j = \nu_i$ (or if $p_e = p_e^{SCC}$), the equilibrium fragmentation is always efficient.

▸ Back

Proposition 1

If $p_e < p_e^{SCC}$*, the weight given to the energy component in the fragmentation decision is insufficient. Therefore, the allocation of production across firms is weakly inefficient.*

Corollary 1

If $p_e = p_e^{SCC}$*, given the allocation of production, the equilibrium allocation of resources within firms is efficient.*
If $p_e = p_e^{SCC}$*, the allocation of production across firms is efficient.*

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Specialization gains vs environmental externality

Suppose $\nu_s = 1$. For $p_e = 0 < p_e^{SCC}$:

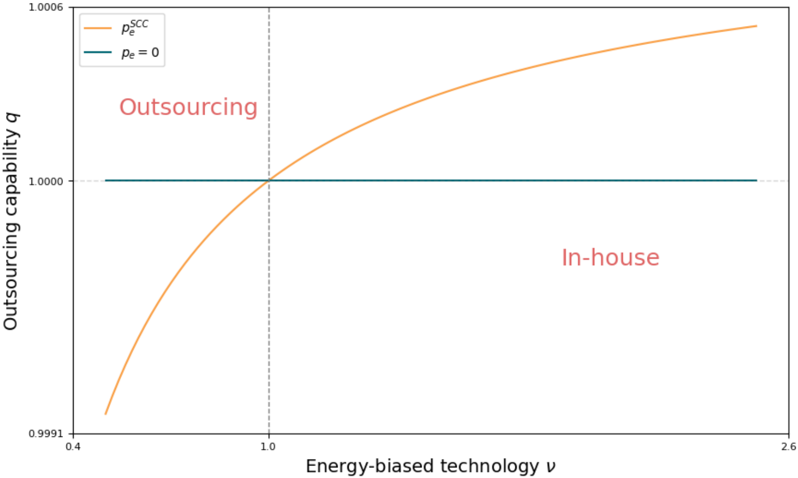

Figure 16: Specialization gains versus energy-biased technology

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Specialization gains vs environmental externality

Suppose $\nu_s = 1$. For $p_e = \infty$:

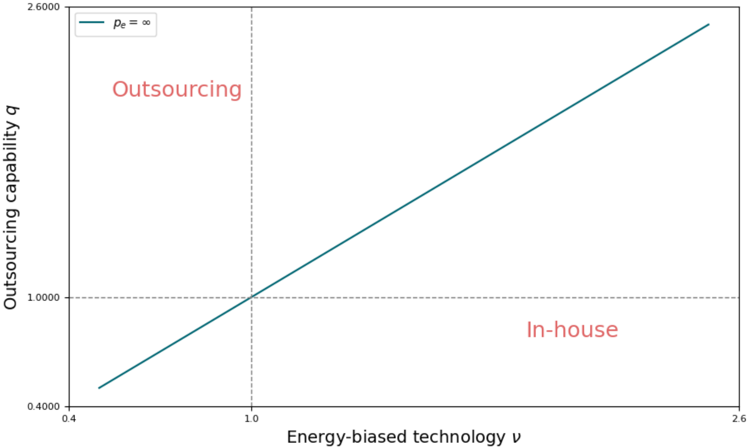

Figure 16: Specialization gains versus energy-biased technology

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Energy intensity

Inefficient fragmentation increases energy intensity

Suppose that i is outsourced if $p_e < p_e^{SCC}$, but i is produced in-house under $p_e = p_e^{SCC}$. With persistent allocation, at $p_e = p_e^{SCC}$:

$$\frac{(e/y)^*}{(e/y)^{\mathcal{SCC}}} = \left(q_{ij}\frac{\nu_i}{\nu_j}\right)^{\frac{\rho}{1-\rho}}\left(\frac{c_{ij}^{\mathcal{O}}/q_{ij}}{c_{ij}^{\mathcal{I}}}\right)^{\frac{1}{1-\rho}}$$

Energy intensity

Inefficient fragmentation increases energy intensity

Suppose that i is outsourced if $p_e < p_e^{SCC}$, but i is produced in-house under $p_e = p_e^{SCC}$. With persistent allocation, at $p_e = p_e^{SCC}$:

$$\frac{(e/y)^*}{(e/y)^{\mathcal{SCC}}} = \left(q_{ij}\frac{\nu_i}{\nu_j}\right)^{\frac{\rho}{1-\rho}} \left(\frac{c_{ij}^{\mathcal{O}}/q_{ij}}{c_{ij}^{\mathcal{I}}}\right)^{\frac{1}{1-\rho}}$$

Inefficient allocation of production implies

- Higher costs $\Rightarrow$ lower output: $c_{ij}^{\mathcal{I}} < \frac{c_{ij}^{\mathcal{O}}}{q_{ij}} \Rightarrow \left(\frac{c_{ij}^{\mathcal{O}}/q_{ij}}{c_{ij}^{\mathcal{I}}}\right)^{\frac{1}{1-\rho}} > 1$

Energy intensity

Inefficient fragmentation increases energy intensity

Suppose that i is outsourced if $p_e < p_e^{SCC}$, but i is produced in-house under $p_e = p_e^{SCC}$. With persistent allocation, at $p_e = p_e^{SCC}$:

$$\frac{(e/y)^*}{(e/y)^{\mathcal{SCC}}} = \left(q_{ij}\frac{\nu_i}{\nu_j}\right)^{\frac{\rho}{1-\rho}} \left(\frac{c_{ij}^{\mathcal{O}}/q_{ij}}{c_{ij}^{\mathcal{I}}}\right)^{\frac{1}{1-\rho}} > 1$$

Inefficient allocation of production implies

- Higher costs $\Rightarrow$ lower output: $c_{ij}^{\mathcal{I}} < \frac{c_{ij}^{\mathcal{O}}}{q_{ij}} \Rightarrow \left(\frac{c_{ij}^{\mathcal{O}}/q_{ij}}{c_{ij}^{\mathcal{I}}}\right)^{\frac{1}{1-\rho}} > 1$
- Higher energy use: $\Rightarrow \left(q_{ij}\frac{\nu_i}{\nu_j}\right)^{\frac{\rho}{1-\rho}} > 1$

Energy intensity

Inefficient fragmentation increases energy intensity

Suppose that i is outsourced if $p_e < p_e^{SCC}$, but i is produced in-house under $p_e = p_e^{SCC}$. With persistent allocation, at $p_e = p_e^{SCC}$:

$$\frac{(e/y)^*}{(e/y)^{SCC}} = \left(q_{ij}\frac{\nu_i}{\nu_j}\right)^{\frac{\rho}{1-\rho}} \left(\frac{c_{ij}^{\mathcal{O}}/q_{ij}}{c_{ij}^{\mathcal{I}}}\right)^{\frac{1}{1-\rho}} > 1$$

Inefficient allocation of production implies

- Higher costs ⇒ lower output: $c_{ij}^{\mathcal{I}} < \frac{c_{ij}^{\mathcal{O}}}{q_{ij}} \Rightarrow \left(\frac{c_{ij}^{\mathcal{O}}/q_{ij}}{c_{ij}^{\mathcal{I}}}\right)^{\frac{1}{1-\rho}} > 1$
- Higher energy use: $\Rightarrow \left(q_{ij}\frac{\nu_i}{\nu_j}\right)^{\frac{\rho}{1-\rho}} > 1$

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Main Take-away

Lower energy intensity requires relatively **energy efficient** firms to produce in-house, but **low energy prices** bias the decision away from $\frac{\nu_j}{\nu_i}$.
The final misallocation depends on the joint distribution of $q_{ij}, \frac{\nu_j}{\nu_i}$.

Energy intensity

Decomposition

$$\Delta log \frac{e_j}{y_j} \approx -\frac{1}{1-\rho}\left(1+\frac{p_e e_k}{wl_k}\right)^{-1}\Delta log p_e$$
$$-\left(\left(\frac{\nu_j}{\nu_i}-1\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_i}{wl_i}\right)^{-1}\right)-\left(1-\frac{q_{jj}}{q_{ij}}\right)\right)\times \mathbf{1}\{\mathcal{O}\rightarrow\mathcal{I}\}$$
$$-\left(\left(1-\frac{\nu_j}{\nu_i}\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_j}{wl_j}\right)^{-1}\right)-\left(\frac{q_{jj}}{q_{ij}}-1\right)\right)\times \mathbf{1}\{\mathcal{I}\rightarrow\mathcal{O}\}$$

Energy intensity

Decomposition

$$\Delta log \frac{e_j}{y_j} \approx -\frac{1}{1-\rho}\left(1+\frac{p_e e_k}{wl_k}\right)^{-1} \Delta log p_e$$

$$-\left(\left(\frac{\nu_j}{\nu_i}-1\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_i}{wl_i}\right)^{-1}\right)-\left(1-\frac{q_{jj}}{q_{ij}}\right)\right)\times \mathbf{1}\{\mathcal{O}\rightarrow \mathcal{I}\}$$

$$-\left(\left(1-\frac{\nu_j}{\nu_i}\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_j}{wl_j}\right)^{-1}\right)-\left(\frac{q_{jj}}{q_{ij}}-1\right)\right)\times \mathbf{1}\{\mathcal{I}\rightarrow \mathcal{O}\}$$

- Direct effect on quantity + reallocation across inputs

Energy intensity

Decomposition

$$\Delta log \frac{e_j}{y_j} \approx -\frac{1}{1-\rho}\left(1+\frac{p_e e_k}{wl_k}\right)^{-1} \Delta log p_e$$
$$-\left(\left(\frac{\nu_j}{\nu_i}-1\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_i}{wl_i}\right)^{-1}\right)-\left(1-\frac{q_{jj}}{q_{ij}}\right)\right)\times \mathbf{1}\{\mathcal{O}\rightarrow\mathcal{I}\}$$
$$-\left(\left(1-\frac{\nu_j}{\nu_i}\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_j}{wl_j}\right)^{-1}\right)-\left(\frac{q_{jj}}{q_{ij}}-1\right)\right)\times \mathbf{1}\{\mathcal{I}\rightarrow\mathcal{O}\}$$

- Direct effect on quantity + reallocation across inputs
- Reallocation across firms from make-or-buy decision (extensive margin)
  - More consideration to energy in the make-or-buy decision ⇒ decreases $\frac{e}{y}$

Energy intensity

Decomposition

$$\Delta log \frac{e_j}{y_j} \approx -\frac{1}{1-\rho}\left(1+\frac{p_e e_k}{wl_k}\right)^{-1}\Delta log p_e$$
$$-\left(\left(\frac{\nu_j}{\nu_i}-1\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_i}{wl_i}\right)^{-1}\right)-\left(1-\frac{q_{jj}}{q_{ij}}\right)\right)\times \mathbf{1}\{\mathcal{O}\rightarrow\mathcal{I}\}$$
$$-\left(\left(1-\frac{\nu_j}{\nu_i}\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_j}{wl_j}\right)^{-1}\right)-\left(\frac{q_{jj}}{q_{ij}}-1\right)\right)\times \mathbf{1}\{\mathcal{I}\rightarrow\mathcal{O}\}$$

- Direct effect on quantity + reallocation across inputs
- Reallocation across firms from make-or-buy decision (extensive margin)
  - More consideration to energy in the make-or-buy decision $\Rightarrow$ decreases $\frac{e}{y}$
  - Joint distribution of $\frac{q_{jj}}{q_{ij}}$ and $\frac{\nu_j}{\nu_i}$

Energy intensity

Decomposition

$$\Delta log \frac{e_j}{py_j} \approx -\frac{1}{1-\rho}\left(1+\frac{p_e e_k}{wl_k}\right)^{-1}\Delta log p_e$$
$$-\left(\left(\frac{\nu_j}{\nu_i}-1\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_i}{wl_i}\right)^{-1}\right)-\left(1-\frac{q_{jj}}{q_{ij}}\right)\right)\times \mathbf{1}\{\mathcal{O}\rightarrow\mathcal{I}\}$$
$$-\left(\left(1-\frac{\nu_j}{\nu_i}\right)\left(1-\frac{1}{1-\rho}\left(1+\frac{p_e e_j}{wl_j}\right)^{-1}\right)-\left(\frac{q_{jj}}{q_{ij}}-1\right)\right)\times \mathbf{1}\{\mathcal{I}\rightarrow\mathcal{O}\}$$

- Direct effect on quantity + reallocation across inputs
- Reallocation across firms from make-or-buy decision (extensive margin)
  - More consideration to energy in the make-or-buy decision ⇒ decreases $\frac{e}{y}$
  - Joint distribution of $\frac{q_{jj}}{q_{ij}}$ and $\frac{\nu_j}{\nu_i}$
  - In the aggregate: distribution of firms "switching" (weighted by sales share) Back

Fixed costs and scale effect in the make-or-buy decision I

Outsourcing capabilities are modeled as fixed costs $\phi_j^{\mathcal{I}}$ and $\phi_j^{\mathcal{O}}$. The make-or-buy decision becomes

$$\pi_j^{\mathcal{O}} - \pi_j^{\mathcal{I}} \lessgtr \phi_j^{\mathcal{O}} - \phi_j^{\mathcal{I}}$$

$$\underbrace{\frac{(\varepsilon-1)^{\varepsilon-1}}{\varepsilon^{\varepsilon}} P^{\varepsilon} \delta_{\omega_j} \left(\frac{Y}{Y_{\omega_j}}\right)^{\frac{\varepsilon}{\eta}} Y_{\omega_j}}_{\equiv \Upsilon_{\omega_j}} \left(c_j^{\mathcal{O}}(\omega_j)^{1-\varepsilon} - c_j^{\mathcal{I}}(\omega_j)^{1-\varepsilon}\right) \lessgtr \phi_j^{\mathcal{O}} - \phi_j^{\mathcal{I}}$$

$$\left(\beta\left(\frac{p_e}{\nu_i}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta) w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)(1-\varepsilon)}{\rho}} - \left(\beta\left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta) w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)(1-\varepsilon)}{\rho}}$$

$$+ \frac{\phi_j^{\mathcal{I}} - \phi_j^{\mathcal{O}}}{\Upsilon_{\omega_j}} \lessgtr 0$$

Fixed costs and scale effect in the make-or-buy decision II

Normalizing $\nu_i = 1$: By how much does the relative energy-biased technology should increase to compensate a fixed cost increase such that the firm's make-or-buy decision is kept fixed?

$$\frac{\partial LHS/\partial log\nu_j}{\partial LHS/\partial log(\phi_j^{\mathcal{I}} - \phi_j^{\mathcal{O}})} = -(\varepsilon - 1)\left(\beta\left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)(1-\varepsilon)}{\rho}} \left(1 + \frac{(1-\beta)}{\beta}\left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}}\right)^{-1} \frac{\Upsilon_{\omega_j}}{\phi_j^{\mathcal{I}} - \phi_j^{\mathcal{O}}}$$

This measure depends on energy prices:

$$\frac{\partial log\left|\frac{\partial LHS/\partial log\nu_j}{\partial LHS/\partial log(\phi_j^{\mathcal{I}} - \phi_j^{\mathcal{O}})}\right|}{\partial log p_e} = -\left(1 + \frac{(1-\beta)}{\beta}\left(\frac{w}{p_e/\nu_j}\right)^{\frac{-\rho}{1-\rho}}\right)^{-1} \times \left[\underbrace{(\varepsilon - 1)}_{\text{Individual scale}} + \underbrace{\frac{\rho}{1-\rho}\frac{(1-\beta)}{\beta}\left(\frac{w}{p_e/\nu_j}\right)^{\frac{-\rho}{1-\rho}}}_{\text{Substitution effect}}\right] + \underbrace{\frac{\partial log P^{\varepsilon} Y}{\partial log p_e}}_{\text{Relative scale}}$$

Fixed costs and scale effect in the make-or-buy decision III

1. **Substitution effect:** Same as in the baseline model - the firms benefit more (less) from a high technology at higher energy prices when $\rho < (>) \, 0$
2. **Individual scale effect:** higher energy prices decrease the firms' profit *level*, which matters for the level impact of a change in ν_j.

Fixed costs and scale effect in the make-or-buy decision IV

Scale effect is *relative*: suppose that $\varepsilon = \eta$ and using that $Y = P^{\frac{1}{\alpha-1}}$

$$\frac{\partial log \frac{\partial LHS/\partial log\nu_j}{\partial LHS/\partial log(\phi_j^{\mathcal{I}} - \phi_j^{\mathcal{O}})}}{\partial log p_e} = \frac{-\rho}{1-\rho}\frac{(1-\beta)}{\beta}\left(\frac{w}{p_e/\nu_j}\right)^{\frac{-\rho}{1-\rho}}\left(1 + \frac{(1-\beta)}{\beta}\left(\frac{w}{p_e/\nu_j}\right)^{\frac{-\rho}{1-\rho}}\right)^{-1}$$
$$+\left(\varepsilon - \frac{1}{1-\alpha}\right)\sum_{\hat{\omega}}\delta_{\hat{\omega}}\int_{k\in\mathcal{N}_{\hat{\omega}}}\left(\frac{\frac{\varepsilon}{\varepsilon-1}c_k(\hat{\omega})}{P}\right)^{1-\varepsilon}\left(1 + \frac{(1-\beta)}{\beta}\left(\frac{w}{p_e/\nu_k}\right)^{\frac{-\rho}{1-\rho}}\right)^{-1} dk$$
$$-(\varepsilon - 1)\left(1 + \frac{(1-\beta)}{\beta}\left(\frac{w}{p_e/\nu_j}\right)^{\frac{-\rho}{1-\rho}}\right)^{-1}$$

With $\rho < (>) \ 0$, relatively large (small) ν_j-firms are relatively less affected. For α not too small:

- **Large ν_j-firms:** The scale effect *reinforces* the inefficiency of the make-or-buy decision at distorted energy prices
- **Small ν_j-firms:** The scale effect *mitigates* the inefficiency of the make-or-buy decision at distorted energy prices

⇒ Distortion larger for large firms ⇒ Quantitative results are a lower bound (No scale effect if comparing q_{jj} and ν_j in the baseline model augmented with identical fixed costs)

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Suppose that $\beta_\omega = \beta\ \forall\omega$, $q_{ij} = q_{jj} = q\ \forall i,j$, $\rho = \gamma$, and that $c_j y_j = \frac{PY}{N}$. Suppose that firms use only one input.

Homogeneous firms: $\nu_j = \nu_i = \nu\ \forall i,j \Rightarrow$ all firms are indifferent between make or buy:

$$\frac{\partial logE/PY}{\partial logp_e} = \underbrace{\frac{-1}{1-\rho}}_{\text{Substitution}} + \underbrace{\frac{\rho}{1-\rho}\left(\frac{p_e}{\nu q}\right)^{\frac{-\rho}{1-\rho}} \beta(c^{el})^{\frac{\rho}{1-\rho}}}_{\text{Scale}} = \frac{-1}{1-\rho} + \frac{\rho}{1-\rho}\frac{p_e E}{PY}$$

Aggregate response

Heterogeneous Firms - No make-or-buy

Suppose that $\beta_\omega = \beta \; \forall \omega$, $q_{ij} = q_{jj} = q \; \forall i, j$, $\rho = \gamma$, and that $c_j y_j = \frac{PY}{N}$. Suppose that firms use only one input.

Heterogeneous Firms - No make-or-buy: $\nu_j \neq \nu_i \; \forall i, j$. Firms $\in \mathcal{F}_I$ produce the input in-house, firms $\in \mathcal{F}_O$ outsource it. $\mathcal{F}_I, \mathcal{F}_O$ are fixed.

$$\frac{\partial logE/PY}{\partial logp_e} = \underbrace{\frac{-1}{1-\rho}}_{\text{Substitution effect}} + \underbrace{\frac{\rho}{(1-\rho)}\left[\int_{j\in\mathcal{F}_I} \frac{e_j}{E}\frac{p_e e_j}{c_j y_j} dj + \int_{j\in\mathcal{F}_O} \frac{e_j}{E}\frac{p_e e_i}{c_i x_i} dj\right]}_{\text{Scale effect}}$$

- ⇒ Hulten-like result
- ⇒ Ex-ante allocation is sufficient

Suppose that $\beta_\omega = \beta\ \forall \omega$, $q_{ij} = q_{jj} = q\ \forall i, j$, $\rho = \gamma$, and that $c_j y_j = \frac{PY}{N}$. Suppose that firms use only one input.

Heterogeneous Firms with make-or-buy. $\mathcal{I}_j$ can vary. Firms in $\mathcal{F}_{OI}$ ($\mathcal{F}_{IO}$) switch from outsourcing (in-house) to in-house (outsourcing):

$$\frac{\Delta E/PY}{E/PY} \approx \frac{-1}{1-\rho}\left(\frac{(E/PY)'}{E/PY}\right)\partial log p_e$$

$$+\frac{\rho}{1-\rho}\left(\int_{j\in\mathcal{F}_{OO}}\left(\frac{e_i}{E}\right)\frac{c_{ij}x_{ij}}{c_i x_i}\frac{p_e e_i}{c_i x_i}dj + \int_{j\in\mathcal{F}_{II}}\left(\frac{e_j}{E}\right)\frac{p_e e_j}{c_j x_j}dj\right)\partial log p_e$$

$$+\int_{j\in\mathcal{F}_{IO}}\left(\frac{e_j}{E}\right)\frac{(c_i^{el})^{\frac{\rho}{1-\rho}} - (c_j^{el})^{\frac{\rho}{1-\rho}}}{(c_j^{el})^{\frac{\rho}{1-\rho}}}dj + \int_{j\in\mathcal{F}_{OI}}\left(\frac{e_i}{E}\right)\frac{(c_j^{el})^{\frac{\rho}{1-\rho}} - (c_i^{el})^{\frac{\rho}{1-\rho}}}{(c_i^{el})^{\frac{\rho}{1-\rho}}}dj$$

$$+\int_{j\in\mathcal{F}_{IO}}\left(\frac{e_i}{E}\right)\frac{\nu_i^{\frac{\rho}{1-\rho}} - \nu_j^{\frac{\rho}{1-\rho}}}{\nu_i^{\frac{\rho}{1-\rho}}}dj + \int_{j\in\mathcal{F}_{OI}}\left(\frac{e_j}{E}\right)\frac{\nu_j^{\frac{\rho}{1-\rho}} - \nu_j^{\frac{\rho}{1-\rho}}}{\nu_i^{\frac{\rho}{1-\rho}}}$$

$\Rightarrow$ Ex-post allocation must be observed or computed

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Aggregate response

Make-or-buy decision required for accurate identification

Suppose that $\beta_\omega = \beta\ \forall \omega$, $q_{ij} = q_{jj} = q\ \forall i,j$, $\rho = \gamma$, and that $c_j y_j = \frac{PY}{N}$. Suppose that firms use **multiple** inputs and produce $\mathcal{I}_j$ (exogenous) in-house.

Heterogeneous firms - fixed make-or-buy: $\nu_j \neq \nu_i\ \forall i,j$, $\mathcal{I}_j$ is heterogeneous but fixed.

$$\frac{\partial log E/PY}{\partial log p_e} = \underbrace{\frac{-1}{1-\rho}}_{\text{Substitution effect}}$$

$$+ \int_j \frac{e_j}{E} \left[\underbrace{\frac{\rho}{1-\rho} \frac{p_e e_j}{|\mathcal{I}_j| c_j y_j}}_{\text{Scale (from energy) effect}} + \underbrace{\frac{-\rho}{(1-\rho)} \frac{\int_{\omega_i \in \mathcal{O}_j} \frac{c_i x_i}{c_j^{el} x_j^{el}} \left(\frac{p_e e_i}{c_i x_i} - \frac{p_e e_j}{|\mathcal{I}_j| c_j y_j} \right) d\omega_i}{1 + \int_{\omega_i \in \mathcal{O}_j} \frac{c_i x_i}{c_j^{el} x_j^{el}} d\omega_i}}_{\text{Scale (from intermediates) effects}} \right] dj$$

Dispersion in $\mathcal{I}_j$ $\Rightarrow$ need to measure $\mathcal{I}_j$ to accurately estimate $\frac{\partial log E/PY}{\partial log p_e}$ Back

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Downstream firm: produces using a continuum Ω of intermediate inputs:

$$y_j = \left(\int_{\omega \in \Omega} x_j(\omega)^\gamma d\omega \right)^{\frac{1}{\gamma}}$$

Make-or-Buy decision: Separate decision for each input. Firm j produces input ω in-house if $c_j^{\mathcal{I}}(\omega) < \frac{c_i^{\mathcal{O}}(\omega)}{q_{ij}}$:

▸ ν_j example

$$\left(\beta_\omega \left(\frac{p_e}{\nu_j} \right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega) w^{\frac{-\rho}{1-\rho}} \right)^{\frac{-(1-\rho)}{\rho}} < q_{ij}^{-1} \left(\beta_\omega \left(\frac{p_e}{\nu_i} \right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega) w^{\frac{-\rho}{1-\rho}} \right)^{\frac{-(1-\rho)}{\rho}}$$

Intermediate inputs $\omega \in \mathcal{I}_j \subseteq \Omega$ are produced in-house while $\omega \in \mathcal{O}_j \subseteq \Omega$ are outsourced.

Energy intensity: Inefficient fragmentation increases energy intensity and decreases welfare $\frac{\partial e/y}{\partial \tilde{i}}_{|\tilde{i}^*} < 0$ and $\frac{\partial W}{\partial \tilde{i}}_{|\tilde{i}^*} > 0$ (while $\frac{\partial W}{\partial \tilde{i}}_{|\tilde{i}^{SCC}} = 0$ by the Envelope Theorem)

▸ Details ▸ Proof ▸ Monotonic ranking

Energy intensity

Firm level

Energy intensity: With $\beta_\omega = \beta \; \forall \omega$ and $\gamma = \rho$, if inputs $\omega \in \mathcal{D}_j \subseteq \mathcal{O}_j$ are inefficiently outsourced:

$$\frac{(e/y)^*}{(e/y)^{SSC}} = \left(\frac{c_j^*}{c_j^{SSC}}\right)^{\frac{1}{1-\rho}} \frac{|\mathcal{I}_j|\nu_j^{\frac{\rho}{1-\rho}} + \int_{\omega \in \mathcal{O}_j} q_{i(\omega)j}^{\frac{\rho}{1-\rho}} \nu_i^{\frac{\rho}{1-\rho}} d\omega}{(|\mathcal{I}_j| + |\mathcal{D}_j|)\nu_j^{\frac{\rho}{1-\rho}} + \int_{\omega \in \mathcal{O}_j \setminus \mathcal{D}_j} q_{i(\omega)j}^{\frac{\rho}{1-\rho}} \nu_i^{\frac{\rho}{1-\rho}} d\omega}$$

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Energy intensity
Firm level

Energy intensity: With $\beta_\omega = \beta \ \forall \omega$ and $\gamma = \rho$, if inputs $\omega \in \mathcal{D}_j \subseteq \mathcal{O}_j$ are inefficiently outsourced:

$$\frac{(e/y)^*}{(e/y)^{SSC}} = \underbrace{\left(\frac{c_j^*}{c_j^{SSC}}\right)^{\frac{1}{1-\rho}}}_{>1} \frac{|\mathcal{I}_j|\nu_j^{\frac{\rho}{1-\rho}} + \int_{\omega \in \mathcal{O}_j} q_{i(\omega)j}^{\frac{\rho}{1-\rho}} \nu_i^{\frac{\rho}{1-\rho}} d\omega}{(|\mathcal{I}_j| + |\mathcal{D}_j|)\nu_j^{\frac{\rho}{1-\rho}} + \int_{\omega \in \mathcal{O}_j \setminus \mathcal{D}_j} q_{i(\omega)j}^{\frac{\rho}{1-\rho}} \nu_i^{\frac{\rho}{1-\rho}} d\omega}$$

Inefficient allocation of production implies

- Higher costs $\Rightarrow$ lower output: $c_j^{\mathcal{I}} < \frac{c_{ij}^{\mathcal{O}}(\omega)}{q_{i(\omega)j}}$ for $i \in \mathcal{D}_j$

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Firm j's energy intensity: With $\beta_\omega = \beta \; \forall \omega$ and $\gamma = \rho$, if inputs $\omega \in \mathcal{D}_j \subseteq \mathcal{O}_j$ are inefficiently outsourced:

$$\frac{(e/y)^*}{(e/y)^{SSC}} = \underbrace{\left(\frac{c_j^*}{c_j^{SSC}}\right)^{\frac{1}{1-\rho}}}_{>1} \underbrace{\frac{|\mathcal{I}_j|\nu_j^{\frac{\rho}{1-\rho}} + \int_{\omega \in \mathcal{O}_j} q_{i(\omega)j}^{\frac{\rho}{1-\rho}} \nu_i^{\frac{\rho}{1-\rho}} d\omega}{(|\mathcal{I}_j| + |\mathcal{D}_j|)\nu_j^{\frac{\rho}{1-\rho}} + \int_{\omega \in \mathcal{O}_j \setminus \mathcal{D}_j} q_{i(\omega)j}^{\frac{\rho}{1-\rho}} \nu_i^{\frac{\rho}{1-\rho}} d\omega}}_{>1} > 1$$

Inefficient allocation of production implies

- Higher costs $\Rightarrow$ lower output: $c_{ij}^{\mathcal{I}} < \frac{c_{ij}^{\mathcal{O}}}{q_{ij}}$ for $i \in \mathcal{D}_j$
- Higher energy use: $(q_{ij}\nu_i)^{\frac{\rho}{1-\rho}} > \nu_j^{\frac{\rho}{1-\rho}}$ for $\omega \in \mathcal{D}_j$

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Energy intensity

Inefficient make-or-buy decision increases energy intensity

For $\beta_\omega = \beta = 0.2$, $\nu_j = 2.15, \nu_i = 1\ \forall i, q_{ij} \sim U[1,3]$ and 100 inputs

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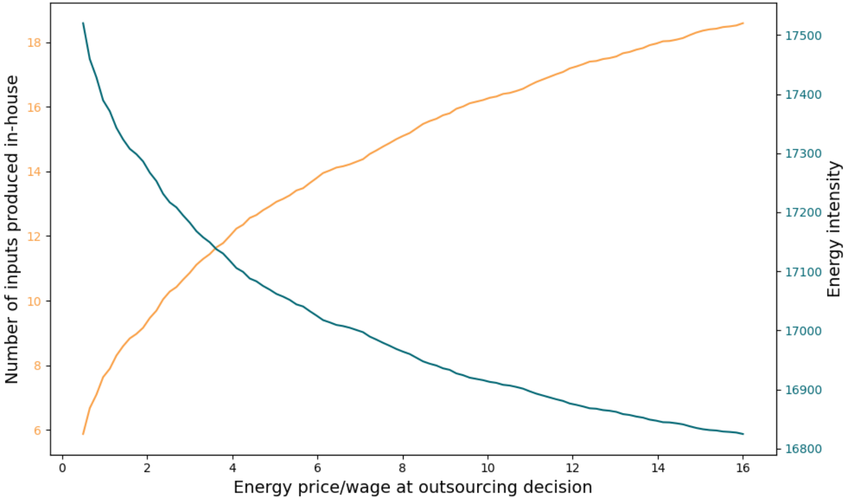

Figure 17: In-house input set, energy intensity and energy prices

Energy intensity

Inefficient fragmentation increases energy intensity

Suppose that $\beta_\omega = \beta \; \forall \omega$, $q_{jj} = 1$, $\nu_i = 1 \; \forall i$, and that intermediates are monotonically ranked by q_{ij}: $\tilde{i} = |\mathcal{I}_j| \Rightarrow c_j^{\mathcal{I}} = \frac{c_{\tilde{i}}^{\mathcal{O}}}{q_{\tilde{i}j}}$.

$$\frac{dlog(e/y)}{dlog\tilde{i}} = \left(\frac{1}{1-\rho} (c_j^{\mathcal{I}})^{\frac{1}{1-\rho}} + \frac{1}{\tilde{i}\nu_j^{\frac{\rho}{1-\rho}} + \int_{\tilde{i}}^{|\Omega|} q_{ij}^{\frac{\rho}{1-\rho}} \, di} \right) \left(\nu_j^{\frac{\rho}{1-\rho}} - q_{\tilde{i}j}^{\frac{\rho}{1-\rho}} \right) < (>) \; 0 \text{ if } \rho < (>) \; 0$$

Energy intensity

Inefficient fragmentation increases energy intensity

Suppose that $\beta_\omega = \beta \; \forall \omega$, $q_{jj} = 1$, $\nu_i = 1 \; \forall i$, and that intermediates are monotonically ranked by q_{ij}: $\tilde{i} = |\mathcal{I}_j| \Rightarrow c_j^{\mathcal{I}} = \frac{c_{\tilde{i}}^{\mathcal{O}}}{q_{\tilde{i}j}}$.

$$\frac{dlog(e/y)}{dlog\tilde{i}} = \left(\frac{1}{1-\rho} (c_j^{\mathcal{I}})^{\frac{1}{1-\rho}} + \frac{1}{\tilde{i}\nu_j^{\frac{\rho}{1-\rho}} + \int_{\tilde{i}}^{|\Omega|} q_{ij}^{\frac{\rho}{1-\rho}} di} \right) \left(\nu_j^{\frac{\rho}{1-\rho}} - q_{\tilde{i}j}^{\frac{\rho}{1-\rho}} \right) < (>) \; 0 \text{ if } \rho < (>) \; 0$$

- With $q_{ij} > 1$ and $\nu_j > 1$, $\tilde{i}^* < (>) \; \tilde{i}^{SCC}$ if $\rho < (>) \; 0$ ▸ Proof ▸ Discrete
  - $\Rightarrow$ Reallocative efficiency: $\frac{dlog(e/y)}{dlogp_e}|_{dc=0} = \frac{dlog(e/y)}{dlog\tilde{i}} \frac{dlog\tilde{i}}{dlogp_e} < 0$
- Similarly, $\frac{\partial W}{\partial \tilde{i}}_{|\tilde{i}^*} \times sgn\left(\frac{d\tilde{i}^*}{dp_e}\right) > 0$ (while $\frac{\partial W}{\partial \tilde{i}}_{|\tilde{i}^{SCC}} = 0$ by the Enveloppe Theorem)

Main Take-away

Lower energy intensity requires relatively **high** ν_j firms to produce in-house, but **low energy prices** at entry bias the decision away from ν_j

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Proof. We have that at the cutoff input $0 < \tilde{i}^* < |\Omega|$:

$$\frac{q_{\tilde{i}^*j}^{\frac{\rho}{1-\rho}} - \nu_j^{\frac{\rho}{1-\rho}}}{1 - q_{\tilde{i}^*j}^{\frac{\rho}{1-\rho}}} = \frac{1-\beta}{\beta}\left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}} > 0$$

With $\rho < (>)\ 0$ and $q_{ij} > 1$, $1 - q_{\tilde{i}^*j}^{\frac{\rho}{1-\rho}} > (<)\ 0$, we must have that $q_{\tilde{i}^*j}^{\frac{\rho}{1-\rho}} - \nu_j^{\frac{\rho}{1-\rho}} > (<)\ 0$.

Energy intensity II

Marginal change in fragmentation

If $p_e < p_e^{SCC}$, we have that at the cutoff input $\tilde{i}^*$:

$$\frac{q_{\tilde{i}^* j}^{\frac{\rho}{1-\rho}} - \nu_j^{\frac{\rho}{1-\rho}}}{1 - q_{\tilde{i}^* j}^{\frac{\rho}{1-\rho}}} = \frac{1-\beta}{\beta}\left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}} > (<) \frac{1-\beta}{\beta}\left(\frac{w}{p_e^{SCC}}\right)^{\frac{-\rho}{1-\rho}} = \frac{q_{\tilde{i}^{SCC} j}^{\frac{\rho}{1-\rho}} - \nu_j^{\frac{\rho}{1-\rho}}}{1 - q_{\tilde{i}^{SCC} j}^{\frac{\rho}{1-\rho}}} \text{ if } \rho < (>)\ 0$$

- If $\rho < (>)\ 0$, $\frac{q_{\tilde{i}j}^{\frac{\rho}{1-\rho}} - \nu_j^{\frac{\rho}{1-\rho}}}{1 - q_{\tilde{i}j}^{\frac{\rho}{1-\rho}}}$ increases (decreases) with $q_{\tilde{i}j}^{\frac{\rho}{1-\rho}}$

$\Rightarrow$ $q_{\tilde{i}^* j} < (>)\ q_{\tilde{i}^{SCC} j} \Rightarrow \tilde{i}^* < (>)\ \tilde{i}^{SCC}$

In marginal products: Intuitively, with $\nu_j > 1$ and $\rho < (>)\ 0$, at $p_e = p_e^{SCC}$ and $\tilde{i}^*$: $MPx_{\tilde{i}^* j}^{\mathcal{I}} < (>)\ MPx_{\tilde{i}^* j}^{\mathcal{O}}$. Since $MPx_{ij}^{\mathcal{O}} \downarrow$ with i, an increase (decrease) in $\tilde{i}^*$ moves the allocation closer to the efficient allocation $MPx_{\tilde{i}j}^{\mathcal{I}} = MPx_{\tilde{i}j}^{\mathcal{O}}$.

Energy intensity III

Marginal change in fragmentation

In welfare: With $W = U(y) - V(e) - D(l)$:

$$\frac{\partial W}{\partial \tilde{i}^*} = U'(y)\frac{\partial y}{\partial \tilde{i}^*} - V'(e)\frac{\partial e}{\partial \tilde{i}} - D'(l)\frac{\partial l}{\partial \tilde{i}^*} = U'(y)\left(p_e\frac{\partial e}{\partial \tilde{i}^*} + w\frac{\partial l}{\partial \tilde{i}^*}\right) - V'(e)\frac{\partial e}{\partial \tilde{i}^*} - D'(l)\frac{\partial l}{\partial \tilde{i}}$$

If the outsourcing allocation was decided at $p_e = p_e^{SCC}$:

$$\frac{\partial W}{\partial \tilde{i}^*} = p\left[\left(p_e^{SCC}\frac{\partial e}{\partial \tilde{i}^*} + w\frac{\partial l}{\partial \tilde{i}^*}\right) - p_e^{SCC}\frac{\partial e}{\partial \tilde{i}^*} - w\frac{\partial l}{\partial \tilde{i}^*}\right] = 0$$

If the outsourcing allocation was decided at $p_e < p_e^{SCC}$:

$$\frac{\partial W}{\partial \tilde{i}^*} = U'(y^*)\left[\left(p_e^{SCC}\frac{\partial e}{\partial \tilde{i}^*} + w\frac{\partial l}{\partial \tilde{i}^*}\right) - \frac{V'(e^*)}{U'(y*)}\frac{\partial e}{\partial \tilde{i}^*} - \frac{D'(l^*)}{U'(y*)}\frac{\partial l}{\partial \tilde{i}^*}\right]$$

$$\Rightarrow sgn(\frac{\partial W}{\partial \tilde{i}^*}) = sgn(\frac{\partial e}{\partial \tilde{i}^*})$$

Since $e^* > e^{SCC}$ and $y^* < y^{SCC}$, $\frac{V'(e^*)}{U'(y*)} < \frac{V'(e^{SCC})}{U'(y^{SCC})} = p_e^{SCC}$. We have that for, $\rho < (>)\ 0$, $\frac{\partial e}{\partial \tilde{i}^*} > (<)\ 0$ and $\tilde{i}^* < (>)\ \tilde{i}^{SCC}$.

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Extensive and intensive margin decomposition

Suppose that $\frac{y_j}{Y} = D\left(\frac{c_j}{P}\right)$, $\gamma = \rho$ and $\beta_\omega = \beta\ \forall\omega$. Then,

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$$\frac{E}{Y} - \left(\frac{E}{Y}\right)^{SCC} =$$

$$\beta \int_{j\in N} \underbrace{D\left(\frac{c_j}{P}\right)\left[\left(\frac{c_j}{p_e}\right)^{\frac{1}{1-\rho}} - \left(\frac{c_j^{SCC}}{p_e^{SCC}}\right)^{\frac{1}{1-\rho}}\right]\left(|\mathcal{I}_j|(\nu_j q_{jj}))^{\frac{\rho}{1-\rho}} + \int_{i:\omega_i\in\mathcal{O}_j}(\nu_i q_{ij})^{\frac{\rho}{1-\rho}}\,di\right)}_{\text{Intensive margin: Within-firm reallocation of energy and labor}}$$

$$+ \underbrace{\left(D\left(\frac{c_j}{P}\right) - D\left(\frac{c_j^{SCC}}{P^{SCC}}\right)\right)\left(\frac{c_j^{SCC}}{p_e^{SCC}}\right)^{\frac{1}{1-\rho}}\left(|\mathcal{I}_j^{SCC}|(\nu_j q_{jj}))^{\frac{\rho}{1-\rho}} + \int_{i:\omega_i\in\mathcal{O}_j^{SCC}}(\nu_i q_{ij})^{\frac{\rho}{1-\rho}}\,di\right)}_{\text{Intensive margin: Across-firm reallocation of household sales shares}}$$

$$+ D\left(\frac{c_j^{SCC}}{P^{SCC}}\right)\left(\frac{c_j^{SCC}}{p_e^{SCC}}\right)^{\frac{1}{1-\rho}}\left(\underbrace{(|\mathcal{I}_j| - |\mathcal{I}_j^{SCC})(\nu_j q_{jj}))^{\frac{\rho}{1-\rho}}}_{\text{Extensive margin: Across-firm reallocation through make-or-buy - in-house input set changes}}\right.$$

$$\left. + \underbrace{\int_{i:\omega_i\in\mathcal{O}_j\&\omega_i\notin\mathcal{O}_j^{SCC}}(\nu_i q_{ij})^{\frac{\rho}{1-\rho}}\,di - \int_{i:\omega_i\notin\mathcal{O}_j\&\omega_i\in\mathcal{O}_j^{SCC}}(\nu_i q_{ij})^{\frac{\rho}{1-\rho}}\,di}_{\text{Extensive margin: Across-firm reallocation through make-or-buy - outsourced input set changes}}\right) dj$$

Full model: supplier selection adds another margin in the outsourced input set changes.

Outsourcing capabilities as a wedge I

Suppose that $\gamma = \rho$, $\beta_\omega = \beta \; \forall \beta$, $\nu_j > 1$ and that we normalize $\nu_i = 1$ and $q_{jj} = 1$.

Cutoff capability. Inputs associated to a $q_{ij} < (>) \; \tilde{q}_j$ are produced in-house (outsourced) by firm j:

$$\tilde{q}_j^{eq} \equiv \left(\frac{\nu_j^{\frac{\rho}{1-\rho}} + \frac{1-\beta}{\beta} \left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}}}{1 + \frac{1-\beta}{\beta} \left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}}} \right)^{\frac{(1-\rho)}{\rho}}$$

Suppose that $q_{ij} \sim U[0, q^m]$ where $q^m > 1$:

$$\frac{e_j}{c_j y_j} = \frac{1}{p_e} \frac{\left(1 + \frac{1-\beta}{\beta} \left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}}\right)^{-1}}{\left(\frac{\tilde{q}_j}{q^m} (\tilde{q}_j^{eq})^{\frac{\rho}{1-\rho}} + (q^m)^{\frac{\rho}{1-\rho}} (1-\rho)(1 - \left(\frac{\tilde{q}_j}{q^m}\right)^{\frac{1}{1-\rho}})\right)} \underbrace{\left(\frac{\tilde{q}_j}{q^m} \nu_j^{\frac{\rho}{1-\rho}} + q_m^{\frac{\rho}{1-\rho}} (1-\rho)(1 - \left(\frac{\tilde{q}_j}{q^m}\right)^{\frac{1}{1-\rho}})\right)}_{\equiv Q_j^e}$$

Outsourcing capabilities as a wedge II

Optimal cutoff. The cutoff $\tilde{q}_j^E$ that minimizes energy intensity is characterized by:

$$\left(\left(\frac{\nu_j}{\tilde{q}_j^{eq}}\right)^{\frac{\rho}{1-\rho}} - \left(\frac{\tilde{q}_j^E}{\tilde{q}_j^{eq}}\right)^{\frac{\rho}{1-\rho}}\right) = \frac{\frac{\tilde{q}_j^E}{q^m}\nu_j^{\frac{\rho}{1-\rho}} + (q^m)^{\frac{\rho}{1-\rho}}(1-\rho)\left(1-\left(\frac{\tilde{q}_j^E}{q^m}\right)^{\frac{1}{1-\rho}}\right)}{\frac{\tilde{q}_j^E}{q^m}(\tilde{q}_j^{eq})^{\frac{\rho}{1-\rho}} + (q^m)^{\frac{\rho}{1-\rho}}(1-\rho)\left(1-\left(\frac{\tilde{q}_j^E}{q^m}\right)^{\frac{1}{1-\rho}}\right)}\left(1-\left(\frac{\tilde{q}_j^E}{\tilde{q}_j^{eq}}\right)^{\frac{\rho}{1-\rho}}\right)$$

$\Rightarrow$ Implicitly determines misallocation $\frac{\tilde{q}_j^E}{\tilde{q}_j^{eq}}$ as a decreasing function of ν_j. Misallocation decreases in p_e.

$\Rightarrow$ Inputs with $q_{ij} \in [\tilde{q}_j^{eq}, \tilde{q}_j^E]$ or $q_{ij} \in [\tilde{q}_j^E, \tilde{q}_j^{eq}]$ could be produced using more energy, but at a higher cost.

Outsourcing capabilities as a wedge III

Missing tax. Reducing misallocation ($\tilde{q}_j^{eq} = \tilde{q}_j^{E}$) requires $\left(\frac{\nu_j}{\tilde{q}_j^{eq}}\right)^{\frac{\rho}{1-\rho}} = 1.$

Downstream firm-specific tax $(1+\tau_j)p_e$ satisfies $1+\tau_j = \nu_j$.

- $\Rightarrow$ ν_j is the firm-specific wedge
- $\Rightarrow$ $\tilde{q}_j^{eq} \neq 1$ is the firm-specific misallocation measure
- $\Rightarrow$ $1+\tau_j = \nu_j \Rightarrow \tilde{q}_j^{eq} = 1$ Allocation solely based on q

Optimal energy intensity. At $\left(\frac{\nu_j/(1+\tau_j)}{\tilde{q}_j^{eq}}\right)^{\frac{\rho}{1-\rho}} = 1$ and $\tilde{q}_j^{E} = 1 \; \forall j$

$$p_e e_{int}^{E} \equiv \left(\frac{p_e e_j}{c_j y_j}\right)^{E} = \left(1 + \frac{1-\beta}{\beta}\left(\frac{w}{p_e}\right)^{\frac{-\rho}{1-\rho}}\right)^{-1}$$

$\Rightarrow$ The optimal allocation equalizes energy intensity across firms.

Outsourcing capabilities as a wedge IV

Misallocation increases energy intensity. $\frac{\tilde{q}_j^E}{\tilde{q}_j^{eq}}$ measures misallocation:

$$\frac{e_j}{c_j y_j} = e^E_{int} \frac{\left(\frac{\nu_j}{\tilde{q}_j^{eq}}\right)^{\frac{\rho}{1-\rho}} + (1-\rho)\left(\left(\frac{\tilde{q}_j^{eq}}{q^m}\right)^{\frac{-1}{1-\rho}} - 1\right)}{\left(1 + (1-\rho)\left(\left(\frac{\tilde{q}_j^{eq}}{q^m}\right)^{\frac{-1}{1-\rho}} - 1\right)\right)}$$

1. If $\tilde{q}_j^E = \tilde{q}_j^{eq}$: no misallocation $\frac{e_j}{c_j y_j} = e^E_{int}$
2. If $\nu_j = 1$: no heterogeneity $\Rightarrow$ no misallocation $\frac{e_j}{c_j y_j} = e^E_{int}$
3. If $\tilde{q}_j^{eq} > 1$, $\frac{\partial log \frac{e_j/c_j y_j}{(e_j/c_j y_j)^*}}{\partial log \tilde{q}_j^{eq}} > 0$. Increasing misallocation increases energy intensity.
4. If $\tilde{q}_j^{eq} < 1$, $\frac{\partial log \frac{e_j/c_j y_j}{(e_j/c_j y_j)^*}}{\partial log \tilde{q}_j^{eq}} < 0$. Decreasing misallocation decreases energy intensity.

Outsourcing capabilities as a wedge V

Aggregate energy intensity. Suppose that $c_j y_j = PY$.

$$\frac{E/PY}{(E/PY)^{\underline{E}}} = \frac{1}{N} \int_j \frac{\left(\frac{\nu_j}{\tilde{q}_j^{eq}}\right)^{\frac{\rho}{1-\rho}} + (1-\rho)\left(\left(\frac{\tilde{q}_j^{eq}}{q^m}\right)^{\frac{-1}{1-\rho}} - 1\right)}{\left(1 + (1-\rho)\left(\left(\frac{\tilde{q}_j^{eq}}{q^m}\right)^{\frac{-1}{1-\rho}} - 1\right)\right)} dj$$

⇒ $\nu_j \neq \nu_i = 1$ induces misallocation, increasing aggregate energy intensity. More (less) dispersion in ν_j increases aggregate energy intensity for $\rho > (<)\ 0$. A higher (lower) correlation between ν_j and q_j^m (trade-off) increases aggregate energy intensity for $\rho > (<)\ 0$.

⚠ PY is maximized in equilibrium since c_j is minimized by all firms

Outsourcing capabilities as a wedge VI

Welfare: Equivalent variation $EV = INC(w, p_e, U^{\underline{E}}) - (PY)^{eq}$

$$EV = (PY)^{\underline{E}} \left(N - \frac{1}{N} \frac{p_e E^{\underline{E}}}{(PY)^{\underline{E}}} \int_j \frac{\tau_j}{1+\tau_j} dj \right) - (PY)^{eq}$$

- EV increases (decreases) with $\tau_j > (<)\ 0$ because it means a larger (smaller) gap between downstream and upstream firms $\Rightarrow$ more misallocation
- EV increases with a MDS in $\tau_j \Rightarrow$ more dispersion $=$ τ_j further from zero $=$ more misallocation

Outsourcing capabilities as a wedge VII

Second best: The social planner chooses the between-firm allocation $\mathcal{A}$ that is characterized by $\mathbf{1}\{i = i_j^*(\omega)\}$:

$$\frac{E}{PY} = \frac{1}{p_e} \int_j \frac{p_j x_j}{PY}(\mathcal{A}) \int_{\omega \in \Omega_j} \int_{i \in \mathcal{S}_\omega \cap \{j\}} \frac{\beta_\omega}{\int_{\hat{\omega} \in \mathcal{I}_i} \beta_{\hat{\omega}}} d\hat{\omega} \frac{c_i x_{ij}(\omega)}{p_j x_j} \frac{p_e e_i}{c_i x_i} \mathbf{1}\{i = i_j^*(\omega)\} di d\omega dj$$

Suppose again that $\gamma = \rho$, $\beta_\omega = \beta$. With a log-linear demand, $p_j y_j = PY$.
$\min_{\mathcal{A}} \frac{E}{PY} = \int_j \int_{\omega \in \Omega_j} \int_{i \in \mathcal{S}_\omega \cap \{j\}} \min_{\mathcal{A}} \frac{c_i x_{ij}(\omega)}{p_j x_j} \frac{p_e e_i}{c_i x_i} \mathbf{1}\{i = i_j^*(\omega)\} di d\omega dj.$

Outsourcing capabilities as a wedge VIII

The allocation is determined by

$$\text{Social Planner} \Rightarrow \min_i \, (q_{ij}\nu_i)^{\frac{\rho}{1-\rho}} \frac{\beta}{1-\beta} \left(\frac{p_e}{w}\right)^{\frac{-\rho}{1-\rho}}$$

$$\text{Equilibrium} \Rightarrow \min_i \, (q_{ij}\nu_i)^{\frac{\rho}{1-\rho}} \frac{\beta}{1-\beta} \left(\frac{p_e}{w}\right)^{\frac{-\rho}{1-\rho}} + \underbrace{q_{ij}^{\frac{\rho}{1-\rho}}}_{Wedge}$$

A relationship-specific tax must account for

- **Intensive margin:** the wedge q_{ij} determines costs and total sales
- **Extensive margin:** producers are perfect substitutes ⇒ allocation is a discrete choice

Outsourcing capabilities as a wedge IX

First best. SP chooses the relationship-specific wedge q_{ij}: $\min_{\{q_{ij}(\omega)\}_{ij\omega}} \frac{E}{PY}$

$$
\begin{aligned}
&\int_j \int_{\omega\in\Omega_j} \int_{i\in\mathcal{S}_\omega\cap\{j\}} \frac{\partial log \frac{p_j x_j}{PY}}{\partial log q_{ij}(\omega)} \frac{\beta_\omega}{\int_{\hat{\omega}\in\mathcal{I}_i} \beta_{\hat{\omega}}} d\hat{\omega} \frac{c_i x_{ij}(\omega)}{p_j x_j} \frac{p_e e_i}{c_i x_i} \mathbf{1}\{i = i_j^*(\omega)\} \\
&+ \frac{p_j x_j}{PY} \frac{\beta_\omega}{\int_{\hat{\omega}\in\mathcal{I}_i} \beta_{\hat{\omega}} d\hat{\omega}} \frac{\partial log \frac{c_i x_{ij}(\omega)}{p_j x_j}}{\partial log q_{ij}(\omega)} \frac{p_e e_i}{c_i x_i} \mathbf{1}\{i = i_j^*(\omega)\} \\
&+ \frac{p_j x_j}{PY} \frac{\beta_\omega}{\int_{\hat{\omega}\in\mathcal{I}_i} \beta_{\hat{\omega}} d\hat{\omega}} \frac{c_i x_{ij}(\omega)}{p_j x_j} \frac{\partial log \frac{p_e e_i}{c_i x_i}}{\partial log q_{ij}(\omega)} \mathbf{1}\{i = i_j^*(\omega)\} \\
&\underbrace{+ \frac{p_j x_j}{PY} \frac{\beta_\omega}{\int_{\hat{\omega}\in\mathcal{I}_i} \beta_{\hat{\omega}} d\hat{\omega}} \frac{c_i x_{ij}(\omega)}{p_j x_j} \frac{p_e e_i}{c_i x_i} \Delta \mathbf{1}\{i = i_j^*(\omega)\}}_{\text{Extensive margin}} \\
&di d\omega dj
\end{aligned}
$$

$\Rightarrow$ the pure technology effect, the intensive allocative effect and the extensive allocative effect

Sustaining within-product and within-input energy intensity heterogeneity

Perfectly substitutable producers require a distortion or another dimension of relationship-specific heterogeneity to sustain within-product and within-input heterogeneity in energy-biased technology: q_{ij}.

$$q_{jf}^{-1}\left(\beta\left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}}+(1-\beta)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}} < q_{if}^{-1}\left(\beta\left(\frac{p_e}{\nu_i}\right)^{\frac{-\rho}{1-\rho}}+(1-\beta)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}$$

and

$$q_{jf'}^{-1}\left(\beta\left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}}+(1-\beta)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}} > q_{if'}^{-1}\left(\beta\left(\frac{p_e}{\nu_i}\right)^{\frac{-\rho}{1-\rho}}+(1-\beta)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}$$

Heterogeneity cannot be sustained if $q_{if} = q_f \ \forall i, f$ or if $q_{if} = q_i \ \forall i, f$.

Today

General production function of input ω:

$$x_{i(\omega)j}(\omega) = q_{i(\omega)j}(\omega) F_\omega \left(\nu_{i(\omega)} e_j, \{x_{k(\hat{\omega})j}(\hat{\omega})\}_{\hat{\omega}\in\Omega_\omega} \right)$$

Cost minimization delivers

$$c_{i(\omega)j}(\omega) = q_{i(\omega)j}(\omega)^{-1} C_\omega \left(\frac{p_e}{\nu_{i(\omega)}}, \{c_{k(\hat{\omega})j}(\hat{\omega})\}_{\hat{\omega}\in\Omega_\omega} \right)$$

Make-or-buy decision (separate for each input):

$$\underbrace{\frac{q_{i(\omega)j}(\omega)}{q_{j(\omega)j}(\omega)} \frac{C_\omega \left(\frac{p_e}{\nu_j}, \{c_{k(\hat{\omega})j}(\hat{\omega})\}_{\hat{\omega}\in\Omega_\omega} \right)}{C_\omega \left(\frac{p_e}{\nu_i}, \{c_{k(\hat{\omega})j}(\hat{\omega})\}_{\hat{\omega}\in\Omega_\omega} \right)}}_{\equiv LHS} \lesseqgtr 1$$

General Framework II

Misallocation

With $\nu_{i(\omega)} = 1$,

$$\frac{\partial log \frac{\partial LHS/\partial log\nu_j}{\partial LHS/\partial log \frac{q_{j(\omega)j}(\omega)}{q_{i(\omega)j}}}}{\partial log p_e} = \frac{\partial^2 log C_\omega \left(\frac{p_e}{\nu_j}, \{c_{k(\hat{\omega})j}(\hat{\omega})\}_{\hat{\omega}\in\Omega_\omega}\right)}{\partial log(\frac{p_e}{\nu_j})^2}$$

$\Rightarrow$ Increases (decreases) with p_e if C_ω is supermodular (submodular) in the effective cost of energy.

$\Rightarrow$ Make-or-buy misallocation across $\nu-$firms, under low p_e, increase energy intensity:

$$\frac{\partial log \frac{p_e e_j}{c_{j(\omega)j}(\omega) x_{j(\omega)j}(\omega)}}{\partial log \nu_j} = -\frac{\partial^2 log C_\omega \left(\frac{p_e}{\nu_j}, \{c_{k(\hat{\omega})j}(\hat{\omega})\}_{\hat{\omega}\in\Omega_\omega}\right)}{\partial log(\frac{p_e}{\nu_j})^2}$$

Back

Quantifying the Effects on Aggregate Energy Intensity I

With identical energy prices and two stages, aggregate energy intensity is

$$\frac{E}{PY} = \frac{1}{p_e} \underbrace{\int_{j \in \mathcal{N}} \underbrace{\frac{p_e e_j}{p_j y_j}}_{\text{Energy exp. shares}} \underbrace{\frac{p_j y_j}{PY}}_{\text{Sales shares}} dj}_{observed}$$

$$= \frac{1}{p_e} \int_{j \in \mathcal{N}^D} \underbrace{\frac{p_j y_j^{hh}}{PY}}_{\text{HH Sales shares}} \left(\underbrace{\int_{\omega \in \mathcal{I}_j} \underbrace{\frac{p_e e_j(\omega)}{p_j(\omega) x_j(\omega)}}_{\text{Input energy exp. shares}} \underbrace{\frac{p_j(\omega) x_j(\omega)}{p_j y_j^{hh}}}_{\text{Input cost share}} d\omega}_{unobserved} \right.$$

$$\left. + \int_{i:\omega_i \in \mathcal{O}_j} \underbrace{\frac{p_e e_i}{p_i x_i}}_{\text{Supplier } i \text{ energy exp. shares}} \underbrace{\frac{p_j(\omega_i) x_j(\omega_i)}{p_j y_j^{hh}}}_{\text{Input cost share}} di \right) dj$$

Quantifying the Effects on Aggregate Energy Intensity II

A small change in energy prices has extensive margin effects. If $p_j y_j^{hh} = PY$:

$$\frac{\Delta E/PY}{E/PY} \approx \Bigg[\underbrace{\left(\int_{\omega \in \mathcal{I}_j \cap \mathcal{I}_j'} \underbrace{\frac{\partial log \frac{p_e e_j(\omega)}{p_j(\omega) y_j(\omega)}}{\partial log p_e}}_{\text{Across primary in } j} \frac{p_j(\omega) y_j(\omega)}{PY} + \frac{p_e e_j(\omega)}{p_j(\omega) y_j(\omega)} \underbrace{\frac{\partial log \frac{p_j(\omega) x_j(\omega)}{p_j y_j}}{\partial log p_e}}_{\text{Across intermediates in } j} d\omega \right)}_{\text{Intensive margin: substitution across inputs}} dlog p_e$$

$$+ \underbrace{\left(\int_{i:\omega \in \mathcal{O}_j \cap \mathcal{O}_j'} \underbrace{\frac{\partial log \frac{p_e e_i}{p_i x_i}}{\partial log p_e}}_{\text{Across primary in } i} \frac{p_j(\omega_i) y_j(\omega_i)}{PY} + \frac{p_e e_i}{p_i x_i} \underbrace{\frac{\partial log \frac{p_j(\omega_i) x_j(\omega_i)}{p_j y_j}}{\partial log p_e}}_{\text{Across intermediates in } j} d\omega \right)}_{\text{Intensive margin: substitution across inputs}} dlog p_e$$

$$+ \underbrace{\int_{\omega_i \in \mathcal{I}_j \& \omega_i \notin \mathcal{I}_j'} \frac{p_e' e_i'}{p_i' x_i'} \frac{p_j(\omega_i)' x_j(\omega_i)'}{p_j' y_j^{hh\prime}} - \frac{p_e e_j(\omega_i)}{p_j(\omega_i) x_j(\omega_i)} \frac{p_j(\omega) x_j(\omega)}{p_j y_j^{hh}} d\omega}_{\text{Extensive margin: reallocation through make-or-buy}}$$

$$+ \underbrace{\int_{\omega_i \notin \mathcal{I}_j \& \omega_i \in \mathcal{I}_j'} \frac{p_e' e_j(\omega_i)'}{p_j(\omega_i)' x_j(\omega_i)'} \frac{p_j(\omega_i)' x_j(\omega_i)'}{p_j' y_j'} - \frac{p_e e_i}{p_i x_i(\omega)} \frac{p_j(\omega_i) x_j(\omega_i)}{p_j y_j^{hh}} d\omega}_{\text{Extensive margin: reallocation through make-or-buy}} \Bigg] dj$$

Quantifying the Effects on Aggregate Energy Intensity III

Heterogeneity in effective energy efficiency $\Rightarrow$ which firm produces ω matters

$\Rightarrow$ No sufficient statistics

- Need both ex-ante and ex-post allocation
- If ex-post allocation is observed, need an exogenous energy price shock
- Need more than micro-elasticity (firm-level intensive margin response) as make-or-buy induces extensive margin (jumps) effects

$\Rightarrow$ Quantifying misallocation requires estimation of "when/where" jumps (reallocation) occur, and their magnitude

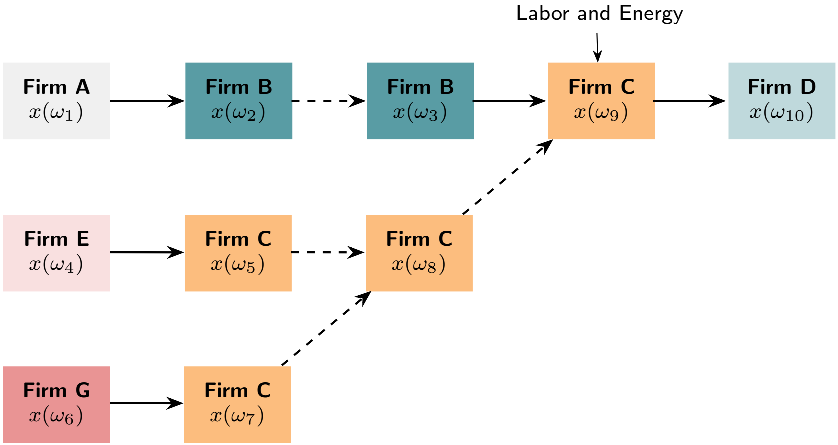
$\Rightarrow$ Quantitative model

Back

Today

M-stage Supply Chains

Make-or-Buy Decisions in a Production Network



▸ Why network? ▸ Back

Make-or-buy decisions in a production network

Setting

Input production: A firm j produces input ω by combining a labor-energy nest:

Details

$$x_j(\omega) = \left(\beta^{1-\rho}(\nu_j e_j(\omega))^\rho + (1-\beta)^{1-\rho} l_j(\omega)^\rho\right)^{\frac{1}{\rho}}$$

Make-or-buy decisions in a production network

Setting

Input production: A firm j produces input ω by combining a labor-energy nest with a set Ω_ω of intermediate inputs: ▸ Details

$$x_j(\omega) = \left(\left(\beta^{1-\rho}(\nu_j e_j(\omega))^\rho + (1-\beta)^{1-\rho} l_j(\omega)^\rho\right)^{\frac{\gamma}{\rho}} + \int_{\hat{\omega}\in\Omega_\omega} (q_{ij} x_{ij}(\hat{\omega}))^\gamma d\hat{\omega} \right)^{\frac{1}{\gamma}}$$

Input production: A firm j produces input ω by combining a labor-energy nest with a set Ω_ω of intermediate inputs:

Details

$$x_j(\omega) = \left(\left(\beta^{1-\rho}(\nu_j e_j(\omega))^\rho + (1-\beta)^{1-\rho} l_j(\omega)^\rho\right)^{\frac{\gamma}{\rho}} + \int_{\hat{\omega}\in\Omega_\omega} (q_{ij} x_{ij}(\hat{\omega}))^\gamma \mathbf{1}\{i = i^*(\omega)\} d\hat{\omega} \right)^{\frac{1}{\gamma}}$$

i is the chosen producer of $\hat{\omega}$ for firm j: $\mathbf{1}\{i = i^*(\omega)\} = \mathbf{1}\{i \in \{\mathcal{S}_{\hat{\omega}} \cap \{j\}\} \cap \mathcal{S}_j^*\}$

- Multiple potential suppliers, inclusive of j: $i \in \{\mathcal{S}_{\hat{\omega}} \cap \{j\}\}$
- Set of chosen producers for firm j: $i \in \mathcal{S}_j^*$

Make-or-buy decisions in a production network

Setting

Input production: A firm j produces input ω by combining a labor-energy nest with a set Ω_ω of intermediate inputs: ▸ Details

$$x_j(\omega) = \left(\left({\beta_\omega}^{1-\rho} (\nu_j e_j(\omega))^\rho + (1-\beta_\omega)^{1-\rho} l_j(\omega)^\rho \right)^{\frac{\gamma}{\rho}} + \int_{\hat{\omega} \in \Omega_\omega} (q_{ij} x_{ij}(\hat{\omega}))^\gamma \mathbf{1}\{i = i^*(\omega)\} d\hat{\omega} \right)^{\frac{1}{\gamma}}$$

Input heterogeneity: β_ω

Make-or-buy decisions in a production network

Setting

Input production: A firm j produces input ω by combining a labor-energy nest with a set Ω_ω of intermediate inputs: ▸ Details

$$x_j(\omega) = \left(\left(\beta_\omega{}^{1-\rho} (\nu_j e_j(\omega))^\rho + (1-\beta_\omega)^{1-\rho} l_j(\omega)^\rho \right)^{\frac{\gamma}{\rho}} + \int_{\hat{\omega} \in \Omega_\omega} (q_{ij} x_{ij}(\hat{\omega}))^\gamma \mathbf{1}\{i = i^*(\omega)\} d\hat{\omega} \right)^{\frac{1}{\gamma}}$$

Make-or-buy decisions in a production network

Setting

Input production: A firm j produces input ω by combining a labor-energy nest with a set Ω_ω of intermediate inputs:

Details

$$x_j(\omega) = \left(\left({\beta_\omega}^{1-\rho} (\nu_j e_j(\omega))^\rho + (1-\beta_\omega)^{1-\rho} l_j(\omega)^\rho \right)^{\frac{\gamma}{\rho}} + \int_{\hat{\omega} \in \Omega_\omega} (q_{ij} x_{ij}(\hat{\omega}))^\gamma \mathbf{1}\{i = i^*(\omega)\} d\hat{\omega} \right)^{\frac{1}{\gamma}}$$

Firm production: Firms can produce each input in-house or outsource it. Its final input ω must be produced in-house ($\omega \in \mathcal{I}_j$):

$$x_j(\omega) = \left((x_j^{\mathcal{I}})^\gamma + (x_j^{\mathcal{O}})^\gamma \right)^{\frac{1}{\gamma}}$$

Input production: A firm j produces input ω by combining a labor-energy nest with a set Ω_ω of intermediate inputs:

Details

$$x_j(\omega) = \left(\left({\beta_\omega}^{1-\rho} (\nu_j e_j(\omega))^\rho + (1-\beta_\omega)^{1-\rho} l_j(\omega)^\rho \right)^{\frac{\gamma}{\rho}} + \int_{\hat{\omega} \in \Omega_\omega} (q_{ij} x_{ij}(\hat{\omega}))^\gamma \mathbf{1}\{i = i^*(\omega)\} d\hat{\omega} \right)^{\frac{1}{\gamma}}$$

Firm production: Firms can produce each input in-house or outsource it. Its final input ω must be produced in-house ($\omega \in \mathcal{I}_j$):

$$x_j(\omega) = \psi_j \left((x_j^{\mathcal{I}})^\gamma + (x_j^{\mathcal{O}})^\gamma \right)^{\frac{1}{\gamma}}$$

TFP $\psi_j \Rightarrow$ Outsourcing capabilities q_{ij} capture purely outsourcing factors

Make-or-buy decisions in a production network

Setting

Input production: A firm j produces input ω by combining a labor-energy nest with a set Ω_ω of intermediate inputs: ▸ Details

$$x_j(\omega) = \left(\left({\beta_\omega}^{1-\rho} (\nu_j e_j(\omega))^\rho + (1-\beta_\omega)^{1-\rho} l_j(\omega)^\rho \right)^{\frac{\gamma}{\rho}} + \int_{\hat{\omega} \in \Omega_\omega} (q_{ij} x_{ij}(\hat{\omega}))^\gamma \mathbf{1}\{i = i^*(\omega)\} d\hat{\omega} \right)^{\frac{1}{\gamma}}$$

Firm production: Firms can produce each input in-house or outsource it. Its final input ω must be produced in-house ($\omega \in \mathcal{I}_j$):

$$x_j(\omega) = \psi_j \left((x_j^{\mathcal{I}})^\gamma + (x_j^{\mathcal{O}})^\gamma \right)^{\frac{1}{\gamma}}$$

Pricing: Firms price at marginal costs. Energy externality is the only distortion.

▸ Summary

Endogenous network: For each input *independently*, firm j chooses to produce in-house or to outsource to the cheapest supplier among the set of *all* potential suppliers $\mathcal{S}_\omega$:

$$c_j(\omega) = \min\left\{ \left(\left(\beta_\omega \left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega) w^{\frac{-\rho}{1-\rho}} \right)^{\frac{(1-\rho)\gamma}{(1-\gamma)\rho}} + \int_{\hat{\omega}\in\Omega_\omega} c_j(\hat{\omega})^{\frac{-\gamma}{1-\gamma}} d\hat{\omega} \right)^{\frac{-(1-\gamma)}{\gamma}}, \min_{i\in\mathcal{S}_\omega} \left(\frac{c_i(\omega)}{q_{ij}}\right) \right\}$$

▸ Margins

Table of Contents

1. Externally calibrate $\eta, \varepsilon, \rho, \gamma, \alpha$.
2. Infer the in-house set $\mathcal{I}_j$ from the recipes Ω and the inputs purchased $\mathcal{O}_j$ ▸ details
3. β_ω: Product fixed effects in $\frac{p_{e_{jt}} e_{jt}}{w_{jt} l_{jt}}$, for the subsample of firms with $\mathcal{I}_j = \{\omega_j\}$ ▸ details
4. Energy-biased technology ν_j: calibrate each ν_j to match

$$\frac{p_e e_j}{w l_j} = \left(\frac{p_e/\nu_j}{w}\right)^{\frac{-\rho}{1-\rho}} \frac{\int_{\hat{\omega}\in\mathcal{I}_j} \beta_{\hat{\omega}} c_j^{el}(\hat{\omega})^{\frac{\gamma-\rho}{(1-\gamma)(1-\rho)}} d\hat{\omega}}{\int_{\hat{\omega}\in\mathcal{I}_j} (1-\beta_{\hat{\omega}}) c_j^{el}(\hat{\omega})^{\frac{\gamma-\rho}{(1-\gamma)(1-\rho)}} d\hat{\omega}}$$

5. Compute $c_j^{el}(\omega) = \left(\beta_\omega \left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega) w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}$ for all firms j and inputs ω

Mapping to the data

Summary

5. Outsourcing capabilities q_{ij}: We face three types of firm-to-firm relationships:
    - 5.1 **Observed relationships**, for which the transaction is directly observed. This is a selected sample;
    - 5.2 **Partially observed relationships**, for which we do not observe the transaction but observe the firms' decision, which gives information on the underlying distribution of q_{ij};
    - 5.3 **Unobserved relationships**, for which we do not observe neither the transaction, nor the decision.

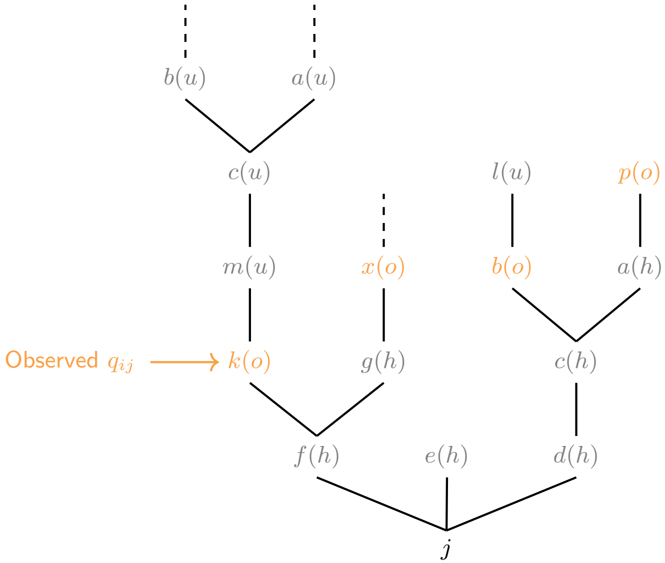
$\Rightarrow$ We want to flexibly model q_{ij} to match its empirical joint distribution with ν_j, ν_i $\Rightarrow$ Buyer and supplier fixed effects

$\Rightarrow$ We estimate the parameters characterizing q_{ij} using the first two groups (no selection), to predict q_{ij} for the third group.

▸ Not DCM ▸ Not hat algebra ▸ Observables

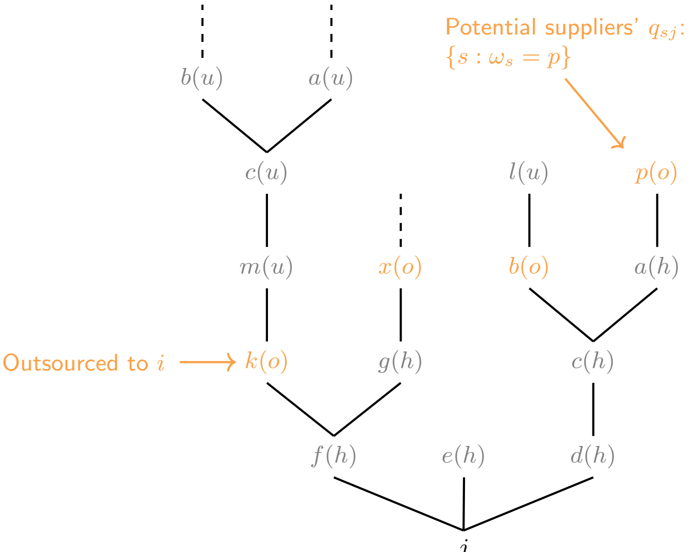
Calibration of the q_{ij}

Step 1: observed pairs - calibrated q_{ij}



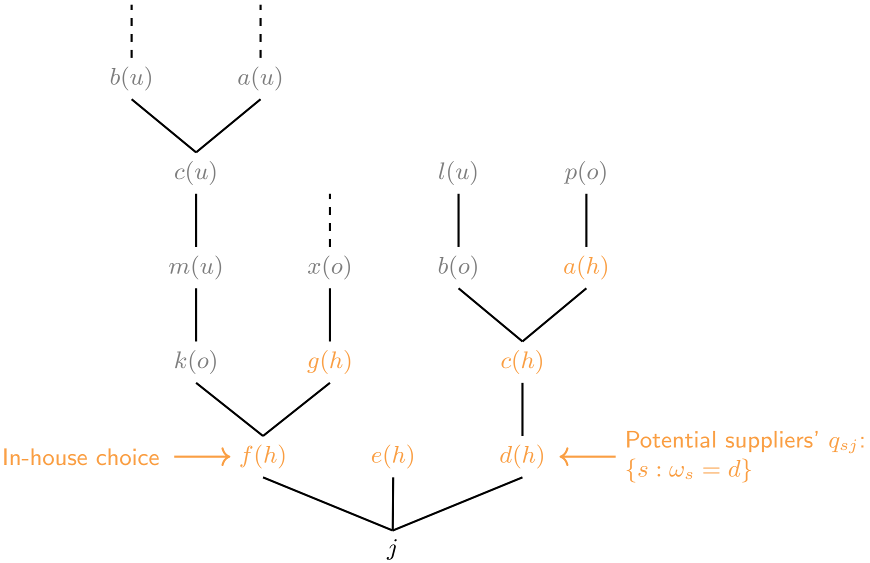
Calibration of the q_{ij}

Step 2: unobserved pairs, observed decision - imputed q_{ij}



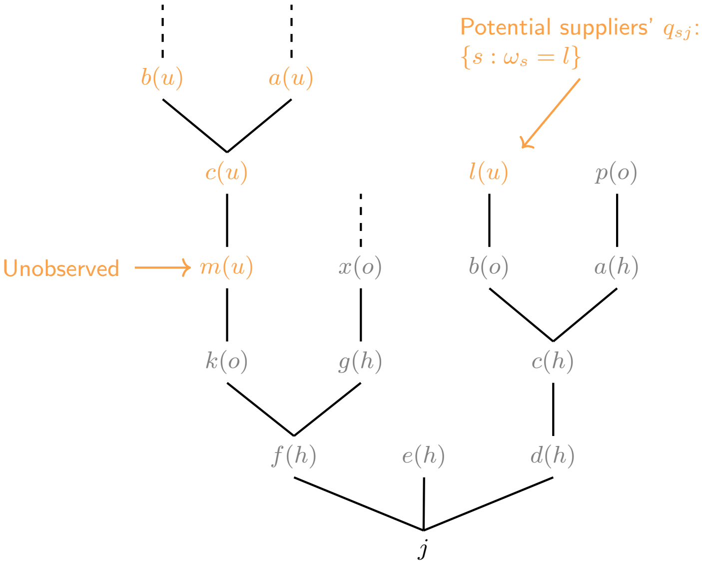
Calibration of the q_{ij}

Step 2: unobserved pairs, observed decision - imputed q_{ij}



Calibration of the q_{ij}

Step 3: unobserved pairs, unobserved decision - predicted q_{ij}



6. Calibrate the distribution of q_{ij}: Suppose $z_{ij} = \frac{1}{q_{ij}} = e^{z_i + z_j + \mathbf{X}_{ij}\delta} e^{\epsilon_{ij}}$ for $i \neq j$ and $q_{jj} = 1$, where $\mathbf{X}_{ij}$ controls for observables (e.g., distance) and $\epsilon_{ij} \sim Exp(\theta)$ is iid across firms and inputs
   $\Rightarrow$ outsourcing capabilities $z_{ij} = \frac{1}{q_{ij}} \sim Pareto\left(e^{z_i^s + z_j^b + \mathbf{X}_{ij}\delta}, \theta\right)$
   - **Outsourced inputs:** For each potential supplier s that was not chosen against supplier i, the conditional distribution is $\frac{1}{q_{sj}} \sim Pareto\left(\frac{c_i(\omega)/q_{ij}}{c_s(\omega)}, \theta\right)$.
   - **In-house inputs:** For each potential supplier s, the conditional distribution is $\frac{1}{q_{sj}} \sim Pareto\left(\frac{c_j^{\mathcal{I}}(\omega)}{c_s(\omega)}, \theta\right)$

$\Rightarrow$ Draw a q_{sj} from those conditional distributions for all firms and inputs for which we observe the decision

7. Estimate the parameters characterizing q_{ij} $(\delta, \{z_i^s\}, \{z_j^b\})$ with PPML on all observed and drawn q_{ij} to build the inputs for which the decision is unobserved.

$\Rightarrow$ All model simulations are data-consistent

Back Wedge

Today

Why do we need a full production network?

Within-input comparison: requires to characterize the buyer and the (potential) supplier in terms of their energy-biased technology.

1. Comparison purely on energy-biased technology $\Rightarrow$ identical production function for all potential suppliers
2. The identification of the energy-biased technology requires knowing the firm boundaries, and in-house production is empirically observed at all stages
3. The set of potential buyers to be compared depends on the set of inputs produced in-house

$\Rightarrow$ In-house set inference requires sequentiality of production

Counterfactual: building the efficient allocation

1. If a firm produces an input in-house, it opens the make-or-buy decision on the upstream inputs
2. Unobserved domestic prices $\Rightarrow$ construct prices recursively

▸ Back

Embedding the model in a production network

1. Misallocation depends on the (potential) suppliers' energy-biased technology
2. Suppliers can also produce in-house
3. Make-or-buy decisions effectively shape the network ▸ Ext. margin ▸ Additional insights

⇒ Challenge: (potential) suppliers' costs, hence make-or-buy decision, affect customers' costs, hence make-or-buy decision

Existing literature:

- Production network literature: single-input firm
- Firm boundary literature: only one stage (firm) makes the outsourcing decision

Except for

- [Boehm and Oberfield, 2023]: tractability from Cobb-Douglas.
- [Eaton et al., 2022]: tractability from assuming different input production functions if produced in-house (primary factor only) or outsourced (tasks combination)
- Endogenous number of suppliers [Huneeus, 2018, Lim, 2018, Acemoglu and Azar, 2020, Bernard et al., 2022, Dhyne et al., 2023, Acemoglu and Tahbaz-Salehi, 2025]: Specialization or love-of-variety gains vs matching fixed costs ⇒ would imply a joint decision.
  ⇒ In-house production as the outside option of not matching (fixed recipes)

Household: Inelastic labor supply: $L^{tot} = 1$. Separable utility from consumption and disutility from energy $U(X, Y, E) = X + \frac{Y^\alpha}{\alpha} + \tilde{U}(E)$, where $X = L^X$ and

$$Y = \left(\sum_\omega \delta_\omega^{\frac{1}{\eta}} Y_\omega^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \text{ and } Y_\omega = \left(\int_{j \in N_\omega} y_{j\omega}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Therefore,

$$X = L^x = 1 - L^m \qquad P_X = w = 1 \qquad PY = Y^\alpha$$

$$P = \left(\sum_\omega \delta_\omega P_\omega^{1-\eta}\right)^{\frac{1}{1-\eta}} \qquad P_\omega = \left(\int_{j \in N_\omega} p_j(\omega)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$$

Input production: A firm j produces input ω by combining a labor-energy nest with a set Ω_ω of intermediate inputs:

$$x_j(\omega) = \left(\left(\beta_\omega^{1-\rho} (\nu_j e_j(\omega))^\rho + (1-\beta_\omega)^{1-\rho} l_j(\omega)^\rho \right)^{\frac{\gamma}{\rho}} + \int_{\hat{\omega} \in \Omega_\omega} (q_{ij} x_{ij}(\hat{\omega}))^\gamma d\hat{\omega} \right)^{\frac{1}{\gamma}}$$

Firm production: Firms can produce each input in-house or outsource it. Its final input ω must be produced in-house ($\omega \in \mathcal{I}_j$):

$$x_j(\omega) = \left((x_j^{\mathcal{I}})^\gamma + (x_j^{\mathcal{O}})^\gamma \right)^{\frac{1}{\gamma}}$$

Pricing: Firms price at marginal costs.

Make-or-Buy Decisions in a production network III

Endogenous network: For each input *independently*, firm j chooses to produce in-house or to outsource to the cheapest supplier among the set of potential suppliers $\mathcal{S}_\omega$:

$$c_j(\omega) = \min\left\{ \left(\left(\beta_\omega \left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega) w^{\frac{-\rho}{1-\rho}} \right)^{\frac{(1-\rho)\gamma}{(1-\gamma)\rho}} + \int_{\hat{\omega}\in\Omega_\omega} c_j(\hat{\omega})^{\frac{-\gamma}{1-\gamma}} d\hat{\omega} \right)^{\frac{-(1-\gamma)}{\gamma}}, \min_{i\in\mathcal{S}_\omega} \left(\frac{c_i(\omega)}{q_{ij}}\right) \right\}$$

Back

Margins for energy (mis)allocation

1. Substitution between energy and labor $\Rightarrow$ driven by elasticity ρ, relative prices $\frac{p_e/\nu_j}{w}$, and energy shares β_ω
2. Substitution between intermediate inputs $(\omega, \hat{\omega})$ $\Rightarrow$ driven by elasticity γ and relative prices $\frac{c_j(\omega)}{c_j(\hat{\omega})}$, where $c_j(\omega) = \min\{c_j^{\mathcal{I}}(\omega), \min_{s \in \mathcal{S}_\omega} \frac{c_s}{q_{sj}}\}$
3. Substitution between *perfectly substitutable* potential suppliers for each input $\Rightarrow$ depends on relative prices $\frac{c_s(\omega)/q_{sj}}{c_i(\omega)/q_{ij}}$ with $\frac{c_s(\omega)}{q_{sj}} = c_j^{\mathcal{I}}(\omega)$ for $s = j$

Higher elasticities of substitution magnify the extent of misallocation [Baqaee and Farhi, 2020]

Back

Multi-dimensional heterogeneity: $k =$firms, $j =$buyers, $i =$suppliers, $\omega = 6$-digit product

1. Energy-biased technology $\nu_k \rightarrow$ large within-product dispersion in energy-biased technology
2. Outsourcing capabilities $q_{ij} = f(z_i, z_j, X_{ij}) \rightarrow$ large within-product dispersion in in-house inputs set + sustain within-product and within-input dispersions in ν with CRS and perfect competition
   2.1 $z_j \rightarrow$ evaluate misallocation from make-or-buy $(\nu_j, \mathcal{I}_j)$
   2.2 $z_i \rightarrow$ evaluate misallocation from supplier selection $(\nu_i, i \in \{\mathcal{S}_j^*\}_j)$ (replace by a supplier fixed cost if systematic empirical correlation between size and number of links, conditional on costs)
3. Input energy share $\beta_\omega \rightarrow$ different energy intensity across inputs
4. TFP $\psi_k \rightarrow q_{ij}$ can be interpreted as outsourcing-specific factors only

Large within-product dispersion in in-house input set:
$\Rightarrow$ production network with make-or-buy decision at each stage

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Today

Additional insights from adding a production network

Additional insights related to the endogenous network literature [Huneeus, 2018, Lim, 2018, Acemoglu and Azar, 2020, Bernard et al., 2022, Dhyne et al., 2023, Acemoglu and Tahbaz-Salehi, 2025]:

- Centrality extends to in-house production ▸ Centrality
- First-order effect of a shock on ν_j or p_e now includes an extensive margin effect if the network is endogenous ▸ Endogenous
  In-house production and outsourcing are inherently perfectly substitutable options
  $\Rightarrow$ Extensive margin effects create discontinuities and potentially large differences in allocation
- Microfoundation of the endogenous network: in-house production as the outside option to changing the number of inputs

▸ Back

Endogenous Production Network I

Insights from endogenous production network:

- Small changes in technology or prices can have discontinuous effects [Acemoglu and Azar, 2020]
- Long-term network adjustments account for 9% and 16% of observed real gross output and welfare changes [Lim, 2018]
- Network adjustment primarily responds to idiosyncratic shocks at the relationship-level rather than at the firm-level [Lim, 2018] $\Rightarrow$ respond to q_{ij} rather than ν_j, ν_i

First-order effect of a shock on ν_j or p_e: With an endogenous production network

1. *Intensive margin:* Summarized by the leontieff inverse (importance through direct and indirect connections)
2. *Extensive margin:* Induce switching from in-house to outsourcing and vice-versa: $\{q_{ij}, \nu_j\} \leftrightarrow \{\nu_i\}$. $\Rightarrow$ effect of a shock on ν_i.

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Quantitative model of in-house production in a production network I

Challenges

Challenges:

- Endogenous make-or-buy decision $\Rightarrow$ endogenous network
- **Mutually exclusive alternatives:** Discrete choice model with asymmetric options
- *Fixed costs to outsource:* joint outsourcing decision for all inputs, but combinatorial discrete choice problem tools [Antras et al., 2017, Arkolakis et al., 2023] cannot be applied
- *Network-dependent decision:* $p_i^{\mathcal{O}}$ depends on $\mathcal{I}_i$

Quantitative model of in-house production in a production network II

Challenges

Existing literature: ▸ Back

- Production network literature: single-input firm
- Firm boundary literature: only one stage (firm) makes the outsourcing decision

Except for

- [Boehm and Oberfield, 2023]: tractability from Cobb-Douglas.
- [Eaton et al., 2022]: tractability from assuming different input production functions if produced in-house (primary factor only) or outsourced (task combination)
- Endogenous number of suppliers [Huneeus, 2018, Lim, 2018, Acemoglu and Azar, 2020, Bernard et al., 2022, Dhyne et al., 2023, Acemoglu and Tahbaz-Salehi, 2025]: Specialization or love-of-variety gains vs matching fixed costs ⇒ would imply a joint decision. ⇒ In-house production as the outside option of not matching (fixed recipes)
- Generic framework with fictitious suppliers [Baqaee and Farhi, 2020]: ex-ante results on allocations requires structure; perfect substitutability of potential suppliers break some results

Today

Mapping to firm-level data I

Overview

Each model firm corresponds to one empirical firm: [Di Giovanni et al., 2024]

- Characterize each firm by its (empirically calibrated) energy-biased technology and its distribution of outsourcing capabilities.

⇒ Replicate the empirical joint distribution of (i) effective energy efficiency and (ii) make-or-buy choices without parametric restrictions.

⚠ Probabilistic specification with two-dimensional firm-level heterogeneity requires inopportune assumptions on the joint distribution of the primitives

⇒ Rich firm-level data allows for exact calibration and a flexible specification of firms' type. Particularly important in our context:

1. **Driver of the results:** Misallocation specifically depends on this joint distribution
2. **No prior:** No empirical knowledge of the relation between effective energy efficiency and make-or-buy decision ⇒ contribution of this paper
3. **Idiosyncratic:** No clear patterns on make-or-buy decisions: highly idiosyncratic factors even across very comparable firms [Antràs, 2020, Bernard and Zi, 2022, Boudreau et al., 2023]
4. **General coverage:** Need to cover all manufacturing sectors to evaluate macroeconomic implications

Mapping to firm-level data II

Overview

- Limitations:
  1. No microfoundation to explain the relation between the make-or-buy decisions and the effective energy efficiency of the firms.

  $\Rightarrow$ Energy-biased technology and outsourcing capabilities are *primitives* of the firms and don't respond to a change in energy price

  2. Calibrated parameters also capture idiosyncratic noises

▸ Back

Firms mostly source an input from a unique supplier

Table 2: Number of Suppliers per Buyer-Input

Statistic	Value
Count	1146164
Mean	2.46
Standard deviation	3.50
Minimum	1
25th percentile	1
Median	1
75th percentile	3
Maximum	336

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Multi-product firms I

- Exclude multi-product firms when building the recipes. Identify multi-product firms:

  1. (baseline) In export data, with distinct product export shares above 5%: $N_{mp} = 15270$ (out of $N = 67093$ manufacturing firms with non-empty recipes)
  2. (hopefully) PRODCOM data for a representative sample of 9000 manufacturing firms
     - 2.1 (robustness) Logit regression on several firms characteristics to predict the probability of being multi-product
  3. (robustness) Likelihood of being multi-product using recipes built with [Fetzer et al., 2024]'s data: are inputs purchased by a firm consistent with the products' recipe?

$\Rightarrow$ Firms identified in robustness steps 1 and 3 are only treated as multi-product firms when building the recipes OR their multiple product portfolio is inferred based on the similarity between its observed input purchases and the recipes of each product.

Multi-product firms II

- In-house set of multi-product firms identified in steps 1 and 2 is the union of the in-house sets of each of its product
- Adjust transactions with multi-product sellers by selecting the product sold by the multi-product seller that is the most likely to enter the recipe of the buyer's product (adjust 23% of the transactions in case 1)
- In the counterfactual: multi-product firms are potential suppliers of all of their products, but all sold at the same price

Multi-product firms III

Table 3: Descriptive Statistics for Multi-Product Firms

	Panel A: Shares of (in %)						
	Manufacturing firms	Sellers	Buyers				
mp. firms	26.14	23.88	20.62				
Transactions involving mp. firms	-	47.99	64.42				
	Panel B: Within Multi-Product Firms						
	Mean	Std. Dev.	Min	P25	Median	P75	Max
Nb. of products per mp. firm	3.29	1.41	2.00	2.00	3.00	4.00	12.00
Product output share	0.32	0.262	0.05	0.10	0.22	0.51	0.95

Notes: A reports the share of multi-product (mp.) firms and of the transactions that involve mp. firms for the final sample of manufacturing firms, and for the sample of manufacturing sellers and buyers in the firm-to-firm transaction data. Panel B reports descriptive statistics for multi-product firms. The first row shows the distribution of the number of products produced by firms operating in more than one product category. The second row reports the distribution of product-level output shares within multi-product firms.

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Methodology I

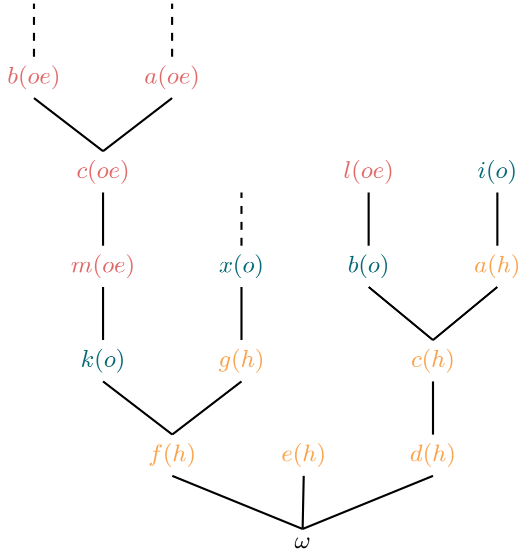

Figure 18: Illustration of the production process of product ω by firm j

Methodology II

Acyclic network: We build an input-output table at the product level that *excludes* cycles, using our firm-to-firm transaction data.

1. *Direct links:* $tr_{ik} = \frac{p^O_{i,k} x^O_{i,k}}{p_k y_k}$ is the average expenditure share of product k on input i. Ignore small transactions ($< 1\%$) to exclude weak or misreported links [Carvalho, 2014, Rachapalli, 2021].
2. *Recipes:* inputs that enter directly or indirectly the production of a product. Exclude cycles by cutting the circling supply chains at products with the lowest transaction shares. ▸ 6-digit
   - 2.1 Direct input set: $\mathcal{U}_j = \{i : tr_{ij} > 0\}$
   - 2.2 Recipe: $\Omega_j = \{i : tr_{ik} tr_{kl} \dots tr_{sj} > 0\}$
   - 2.3 Products that can be produced, directly or indirectly, using input j: $\mathcal{D}_j = \{i : tr_{jk} tr_{kl} \dots tr_{si} > 0\}$
3. Outsourced input set: $\mathcal{O}_j = \{l : tr_{lj} > 0$ and $p_{lj} x_{lj} > 0\}$.

Methodology III

The set of inputs produced in-house by a firm producing product j is the intersection between the set of inputs that are upstream to product ω ($\mathcal{U}_\omega$) and the set of products that are downstream the products that are purchased by the firms ($\cup_{\hat{\omega}\in\mathcal{O}_j}\mathcal{D}_{\hat{\omega}}$).
Formally,

$$\mathcal{I}_j = \mathcal{U}_\omega \ \cup_{\hat{\omega}\in\mathcal{O}_j} \mathcal{D}_{\hat{\omega}} \tag{1}$$

Alternatively, we can also build

$$\mathcal{I}_j = \mathcal{U}_j \setminus \left(\mathcal{O}_j \ \cup_{\hat{\omega}\in\mathcal{O}_j} \mathcal{U}_{\hat{\omega}}\right) \tag{2}$$

Limitations: inputs being used in multiple supply chains

- Equation (1) includes $k(o), m(oe)$ in $\mathcal{I}_j$ due to $b(o)$
- Equation (2) exclude $c(h)$ from $\mathcal{I}_j$ due to $c(oe)$

Recursive iterations: Move upstream and consider direct links only:

1. Start from product j and consider all direct inputs of product j ($\mathcal{U}_\omega = \{\hat{\omega} : tr_{\hat{\omega},\omega} > 0\}$: $\hat{\omega} \in \mathcal{I}_j$) if $\hat{\omega} \notin \mathcal{O}_j$
2. Move upstream: in-house inputs $\hat{\omega} \in \mathcal{I}_j$ become the "products", and repeat step 1 for each $\hat{\omega} \in \mathcal{I}_j$.
3. Move upstream until $\mathcal{I}_j$ is left unchanged.

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Recipes I

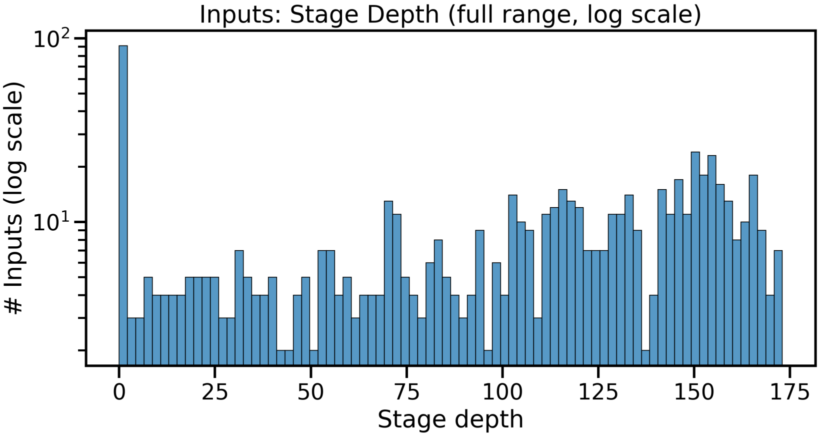

Figure 19: Distribution of products by stage depth

Notes: The graph plots the number of products at each stage depth, which is measured as the number of inputs between the product itself and its most upstream input. ▸ Back

Recipes II

Table 4: Descriptive Statistics: Product Recipes

	Statistics							
	Nb. of obs.	Mean	Std. Dev.	Min	P25	Median	P75	Max
Number of inputs in recipes	1739	353.52	585.29	0	0	0	862	1620
Number of direct inputs in recipes	1739	1.62	1.48	0	1	1	2	15
Number of direct downstream products	1739	1.29	1.33	0	1	1	2	12
Stage depth	1739	24.8	44.46	0	0	0	29	141

Notes: This table reports descriptive statistics on the number of inputs entering the recipe of the products (first row), the number of direct inputs of a product (second row), the number of product that directly use the product as an input (third row) and the stage depth (fourth row), that is, the longest path from any upstream input to the product. Inputs and products are classified at the 6-digit ATECO level. Back

Overlap of recipes within 4-digit code

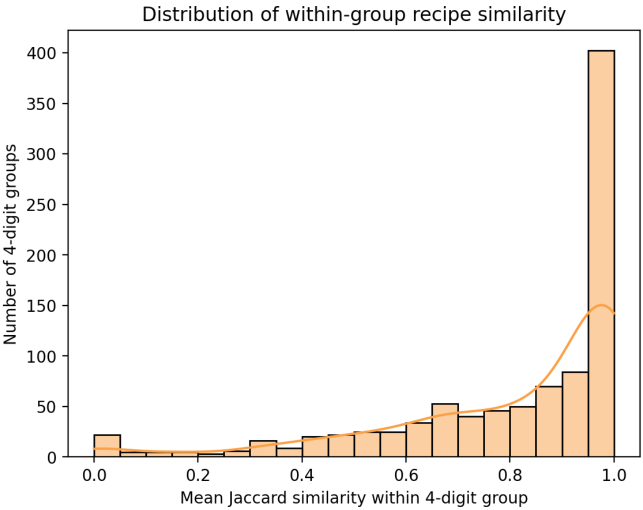

Figure 20: Distribution of within-4-digit codes recipe similarity

Notes: The graph plots the distribution of the within HS 4-digit cpde average Jaccard similarity index between the recipes of the HS 6-digit products. Recipes are built using the production network of [Fetzer et al., 2024]. ▸ Back

In-house Production I

Descriptive Statistics

Table 5: Descriptive Statistics: In-House Input Production

	Statistics						
	Mean	Std. Dev.	Min	P25	Median	P75	Max
Number of in-house inputs	24.87	40.61	0	1	8	20	183
Share of in-house inputs in recipe	0.036	0.091	0	0.0008	0.006	0.02	1.0

Notes: This table reports descriptive statistics on the number of inputs produced in-house by the manufacturing firms in the sample. In-house inputs are inferred by the authors from the products' recipes and the firms' input expenditures.

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In-house Production II

Descriptive Statistics

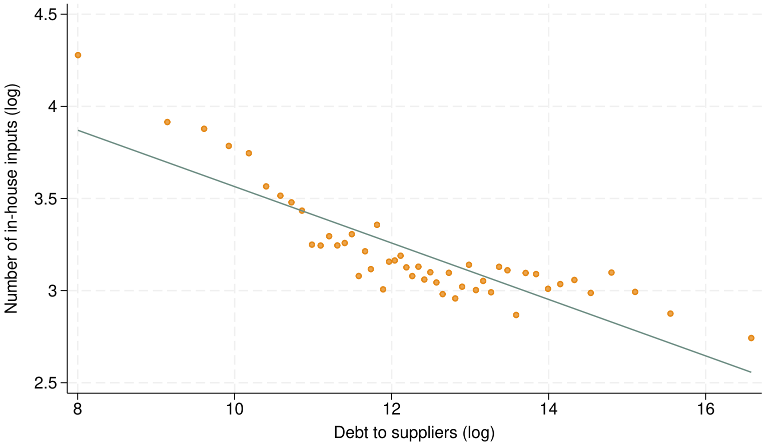

Figure 21: Correlation between number of inputs produced in-house and debt to suppliers

Notes: The correlation is residualized on product (6 digits) and region fixed effects, for the sample of single-product Italian manufacturing firms active in 2019.

▸ Back

Energy-biased technology

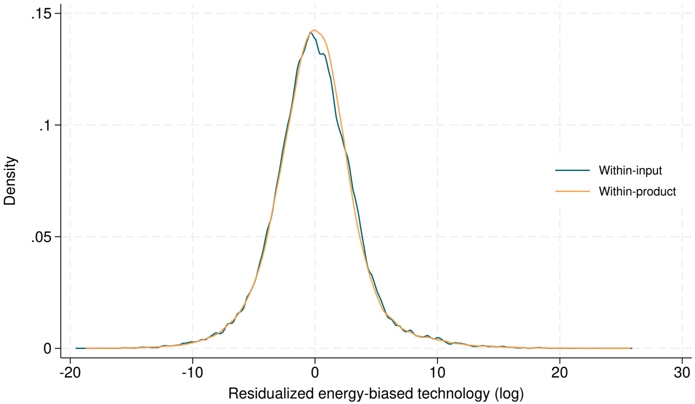

Figure 22: Dispersion in residualized energy-biased technology within input and product

Notes: The residuals are obtained from a cross-sectional (2019) regression on input and core product (6 digits) fixed effects, the number of inputs produced in-house, and factor prices, for the sample of single-product Italian manufacturing firms. Within-product only considers firms for which the input is their core product. Energy-biased technology is calibrated based as the residuals in energy intensity

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Do energy efficient firms outsource more? I

$$log\frac{p_{ij}^{\mathcal{O}}(\omega)x_{ij}^{\mathcal{O}}(\omega)}{p_j(\omega_j)y_j(\omega_j)} = \frac{\gamma}{1-\gamma}log(p_j(\omega_j)) - \frac{\gamma}{1-\gamma}log(c_i^{\mathcal{O}}(\omega)) + \frac{\gamma}{1-\gamma}logq_{ij}$$

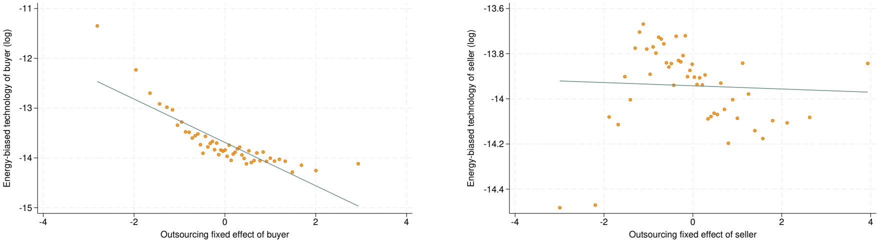
Regress: $$log\frac{p_{ij}^{\mathcal{O}}(\omega)x_{ij}^{\mathcal{O}}(\omega)}{p_j(\omega_j)y_j(\omega_j)} = z_i + z_j + z_\omega + z_{\omega_j} + \mathbf{X}'_{ij}\delta \quad (3)$$

$$\Rightarrow \frac{\gamma}{1-\gamma}logq_{ij} = \hat{z}_i + \hat{z}_j - \left(\frac{\gamma}{1-\gamma}log(\mu_j) - \frac{\gamma}{1-\gamma}log(\mu_i)\right) + \mathbf{X}'_{ij}\hat{\delta} + \varepsilon_{ij}$$

Assume ε_{ij} iid, selection only on observables and $\mu_j = \mu_i = \mu \; \forall i, j$

Do energy efficient firms outsource more? II

With $\gamma < 0$, energy efficient buyers outsource *more*, to (weakly) more energy efficient suppliers.



(a) Buyer (b) Seller

Figure 23: Correlation between ν_j and outsourcing capability fixed effects

Notes: The figures plot the correlation between the calibrated energy-biased technology (ν) of the buyer (left) and seller (right) and its naive outsourcing fixed effects as estimated in equation (3). ▸ Back

Today

Empirical moments characterizing outsourcing

Value-added

VA to sales ratio is a sufficient statistic for outsourcing, *conditional* on intermediate input effective costs and energy-biased technology (suppose $\gamma = \rho$ and $\beta_\omega = \beta \ \forall \omega$):

$$\frac{VA}{sales} = \frac{wl_j}{c_j(\omega_j)x_j(\omega_j)} = \left(\frac{w}{c_j(\omega_j)}\right)^{\frac{-\rho}{1-\rho}} |\mathcal{I}_j|(1-\beta)$$

$$= \frac{(1-\beta)w^{\frac{-\rho}{1-\rho}}}{\left(\beta\left(\frac{p_e}{\nu_j}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta)w^{\frac{-\rho}{1-\rho}}\right) + \frac{1}{|\mathcal{I}_j|}\int_{\hat{\omega}\in\mathcal{O}_j}\left(\frac{c_i(\hat{\omega})}{q_{ij}}\right)^{\frac{-\rho}{1-\rho}} d\hat{\omega}}$$

Large $\frac{VA}{sales}$ might have

1. Large in-house input production (large $|\mathcal{I}_j|$)
2. Low intermediate input costs (large q_{ij}) $\Rightarrow$ small in-house input production (small $|\mathcal{I}_j|$)

$\Rightarrow$ ambiguous raw correlation between $\frac{VA}{sales}$ and $\mathcal{I}_j$

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Calibrating the product-specific parameter I

1. **Strategy 1:** Energy expenditure share of each four-digit input from the US BEA input-output tables

2. **Strategy 2:** For firms with $\mathcal{I}_j = \{\omega_j\}$, regress

$$log \frac{p_e e_j}{w l_j} = \underbrace{log \frac{\beta_\omega}{1 - \beta_\omega} \mathbf{1}\{\mathcal{I}_j = \{w\}\}}_{\omega \text{ FE}} + \frac{\rho}{1-\rho} log\left(\frac{w}{p_e}\right) + \underbrace{\frac{\rho}{1-\rho} log(\nu_j)}_{\text{residuals}}$$

Instrumenting using national fuel prices interacted with fuel shares, and with $\hat{\omega} \in \mathcal{O}_j$ foreign suppliers' prices to account for selection following [Ahn and Powell, 1993]. Use panel if not enough $\mathcal{I}_j = \{\omega_j\}$ firms for each ω_j.

⚠ Need sufficiently many $\mathcal{I}_j = \{\omega_j\}$ firms for each $\omega_j \Rightarrow$ use time series $(2019 - 2023)$

⚠ (Cannot identify ν_j as firm FE because no within-firm variations in product set.)

$\Rightarrow$ Can also estimate $\rho(\omega)$ by interacting $log\left(\frac{w}{p_e}\right)$ and $\mathbf{1}\{\mathcal{I}_j = \{w\}\}$

Calibrating the product-specific parameter II

3. **Strategy 3:** Non-linear AKM regression [Abowd et al., 1999] on firm and product fixed effects, restricting to single-product firms, and controlling for energy and labor prices - leverage variations in $\mathcal{I}_j$, across firms

⇒ **Robustness:** leverage time variations - Non-linear AKM regression [Abowd et al., 1999] on firm, product and time fixed effects, restricting to single-product firms, and controlling for energy and labor prices and important changes in fuel mix.

- ⚠ Short panel: $2019 - 2023$ (excluding COVID year 2021)
- ⚠ Energy-biased technology must be fixed over time.

Calibrating the product-specific parameter III

4. **Strategy 4:** Products differ in terms of the elasticity of substitution $\rho(\omega)$:

$$\frac{p_e e_j}{w l_j} = \frac{\int_{\omega \in \mathcal{I}_j} \left(\frac{c_j(\omega_j)}{c_j^{el}(\omega)}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{c_j^{el}}{p_e/\nu_j}\right)^{\frac{\rho(\omega)}{1-\rho(\omega)}} d\omega}{\int_{\omega \in \mathcal{I}_j} \left(\frac{c_j(\omega_j)}{c_j^{el}(\omega)}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{c_j^{el}}{w}\right)^{\frac{\rho(\omega)}{1-\rho(\omega)}} d\omega}$$

Use single-product firms with $\mathcal{I}_j = \{\omega_j\}$,

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$$\Delta log \frac{p_{e_{jt}} e_{jt}}{w_{jt} l_{jt}} = \frac{\rho(\omega)}{1-\rho(\omega)} \Delta log \left(\frac{w_{jt}}{p_{e_{jt}}}\right) \mathbf{1}\{\mathcal{I}_j = \{w\}\} + \underbrace{\frac{\rho(\omega)}{1-\rho(\omega)} \Delta log \nu_j}_{=0} + \delta \mathbf{1}\{\text{Fuel changes}\} + \eta_{jt}$$

Instrumenting using national fuel prices interacted with fuel shares. FD to partial out ν_j (controlling for fuel mix changes) ⇒ neutralize selection

⇒ **Robustness:** Using firm×input level survey (EAP) of a representative sample of French firms $(2010-2020)$ and firm-level fuel costs and quantity from EACEI - Infer $\mathcal{I}_j$ from Italian recipes and input purchases, and use a shift-share instrument as in [Marin and Vona, 2021, Fontaine and Marullaz, 2024]

⇒ **For now:** Use [Bretschger and Jo, 2021b] estimates for 19 NACE 2 industries

Inconsistencies in Strategy 1 (BEA):

$$\frac{p_e E(\omega)}{wL(\omega)} = \left(\frac{p_e}{w}\right)^{\frac{-\rho}{1-\rho}} \frac{\beta_\omega \int_{j\in\mathcal{N}(\omega)} \nu_j^{\frac{\rho}{1-\rho}} c_j(\omega) x_j(\omega) c_j^{el}(\omega)^{\frac{\rho}{1-\rho}} \left(\frac{c_j(\omega)}{c_j^{el}(\omega)}\right)^{\frac{\gamma}{1-\gamma}} dj + \int_{j\in\mathcal{N}(\omega)} \nu_j^{\frac{\rho}{1-\rho}} c_j(\omega)x}{(1-\beta_\omega) \int_{j\in\mathcal{N}(\omega)} c_j(\omega) x_j(\omega) c_j^{el}(\omega)^{\frac{\rho}{1-\rho}} \left(\frac{c_j(\omega)}{c_j^{el}(\omega)}\right)^{\frac{\gamma}{1-\gamma}} dj + \int_{j\in\mathcal{N}(\omega)} c_j(\omega) x_j(\omega)}$$

Firm-level heterogeneity $(\nu_j, q_{ij} \to \mathcal{I}_j)$ biases $\frac{\beta_\omega}{1-\beta_\omega}$.

Calibrating the product-specific parameter V

Challenges in Strategy 3 (AKM): Highly non-linear estimation - comparing:

$$
\begin{aligned}
& log\frac{p_{e_j}e_j}{w_j l_j} - log\frac{p_{e_i}e_i}{w_i l_i} \\
& = \underbrace{\frac{\rho}{1-\rho}\left(log\left(\frac{w_j}{p_{e_j}}\right) - log\left(\frac{w_i}{p_{e_i}}\right)\right)}_{\text{Controls}} + \frac{\rho}{1-\rho}\underbrace{(log(\nu_j) - log(\nu_i))}_{\text{Firm FE}} \\
& \underbrace{+ log\left(\int_{\hat{\omega}\in\mathcal{I}_j}\beta_{\hat{\omega}}c_j^{el}(\hat{\omega})^{\frac{\gamma-\rho}{(1-\gamma)(1-\rho)}}d\hat{\omega}\right) - log\left(\int_{\hat{\omega}\in\mathcal{I}_j}(1-\beta_{\hat{\omega}})c_j^{el}(\hat{\omega})^{\frac{\gamma-\rho}{(1-\gamma)(1-\rho)}}d\hat{\omega}\right)}_{\text{Firm } j \text{ FE, controls and } \hat{\omega} \text{ FE}} \\
& \underbrace{- log\left(\int_{\hat{\omega}\in\mathcal{I}_i}\beta_{\hat{\omega}}c_i^{el}(\hat{\omega})^{\frac{\gamma-\rho}{(1-\gamma)(1-\rho)}}d\hat{\omega}\right) + log\left(\int_{\hat{\omega}\in\mathcal{I}_i}(1-\beta_{\hat{\omega}})c_i^{el}(\hat{\omega})^{\frac{\gamma-\rho}{(1-\gamma)(1-\rho)}}d\hat{\omega}\right)}_{\text{Firm } i \text{ FE, controls and } \hat{\omega} \text{ FE}}
\end{aligned}
$$

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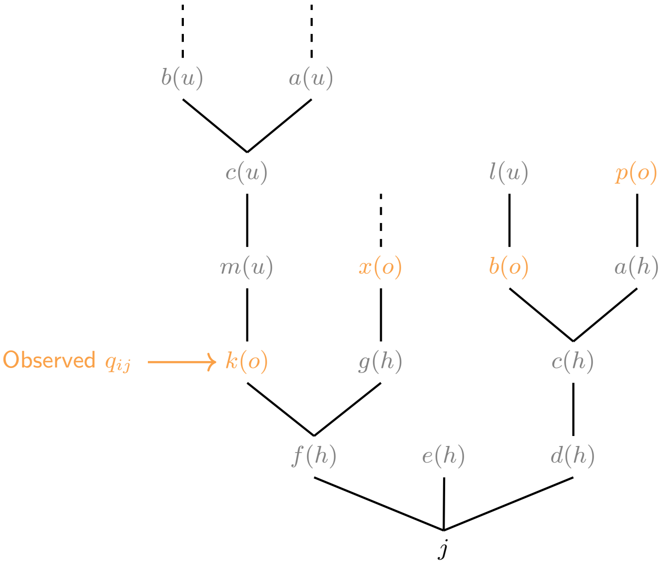
Calibration of the q_{ij} I

Step 1: observed pairs - calibrated q_{ij}



Calibration of the q_{ij} II

Step 1: observed pairs - calibrated q_{ij}

1. Compute $c_i(\omega)$ for all firms recursively, starting from the terminal nodes. Define the set $\mathcal{R}$ as the set of inputs for which we have identified all suppliers' c_i:

   1.1 Using all the transactions of input $\omega \in \mathcal{R}$,

$$log(q_{ij}) = \frac{1-\gamma}{-\gamma}\left(log\frac{p_e e_j + wl_j}{p^O_{ij}x^O_{ij}} + \frac{\gamma}{1-\gamma}log\left(\frac{c_j^{el,tot}}{c_i(\omega)}\right)\right)$$

   1.2 Move downstream to the firms that only purchase inputs $\omega \in \mathcal{R}$.

$$c_j(\omega_j) = c_j^{\mathcal{I}}(\omega_j) = \left(c_j^{el}(\omega_j)^{\frac{-\gamma}{(1-\gamma)}} + \int_{\hat{\omega}\in\Omega_{\omega_j}\&\hat{\omega}\in\mathcal{I}_j} c_j^{el}(\hat{\omega})^{\frac{-\gamma}{(1-\gamma)}} d\hat{\omega} + \int_{\hat{\omega}\in\Omega_{\omega_j}\subseteq\mathcal{R}\&\hat{\omega}\in\mathcal{O}_j}\left(\frac{c_{\hat{\omega}}}{q_{ij}}\right)^{\frac{-\gamma}{1-\gamma}} d\hat{\omega}\right)^{\frac{-(1-\gamma)}{\gamma}}$$

   If $c_j(\omega_j)$ identified for all producers of $\omega_j \Rightarrow \omega_j \in \mathcal{R}$

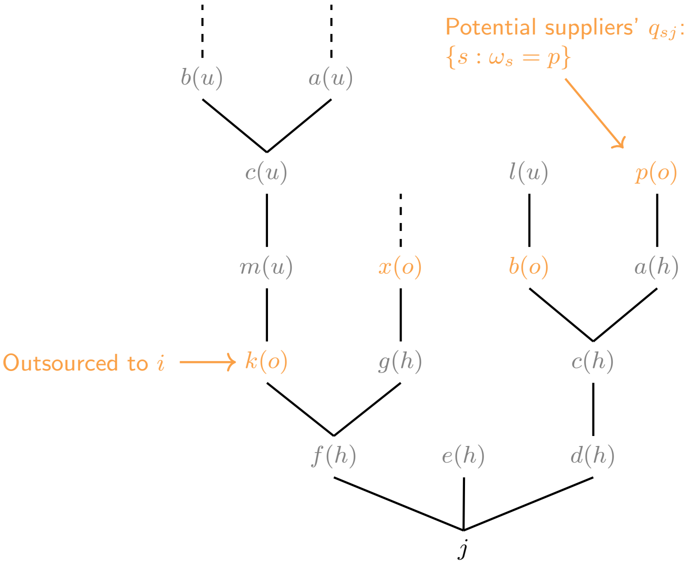
   1.3 Repeat steps 1 and 2 until we have identified all unit costs for all suppliers.

2. Finally, for all for all $\omega \in \mathcal{I}_j\ \forall j$:

$$c_j^{\mathcal{I}}(\omega) = \left(c_j^{el}(\omega)^{\frac{-\gamma}{(1-\gamma)}} + \int_{\hat{\omega}\in\Omega_{\omega}\&\hat{\omega}\in\mathcal{I}_j} c_j^{el}(\hat{\omega})^{\frac{-\gamma}{(1-\gamma)}} d\hat{\omega} + \int_{\hat{\omega}\in\Omega_{\omega}\&\hat{\omega}\in\mathcal{O}_j}\left(\frac{c_{\hat{\omega}}}{q_{ij}}\right)^{\frac{-\gamma}{1-\gamma}} d\hat{\omega}\right)^{\frac{-(1-\gamma)}{\gamma}}$$

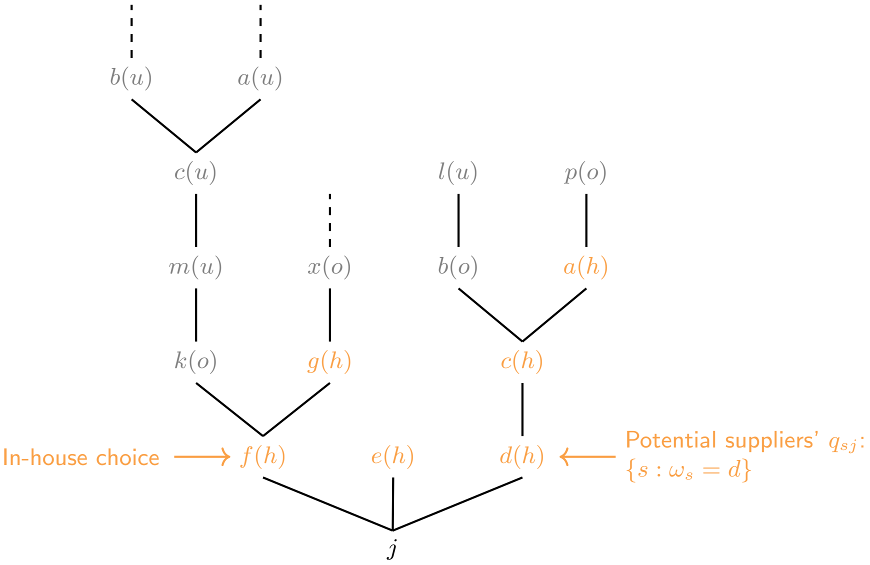
Calibration of the q_{ij} I

Step 2: unobserved pairs, observed decision - imputed q_{ij}



Calibration of the q_{ij} II

Step 2: unobserved pairs, observed decision - imputed q_{ij}



Calibration of the q_{ij} III

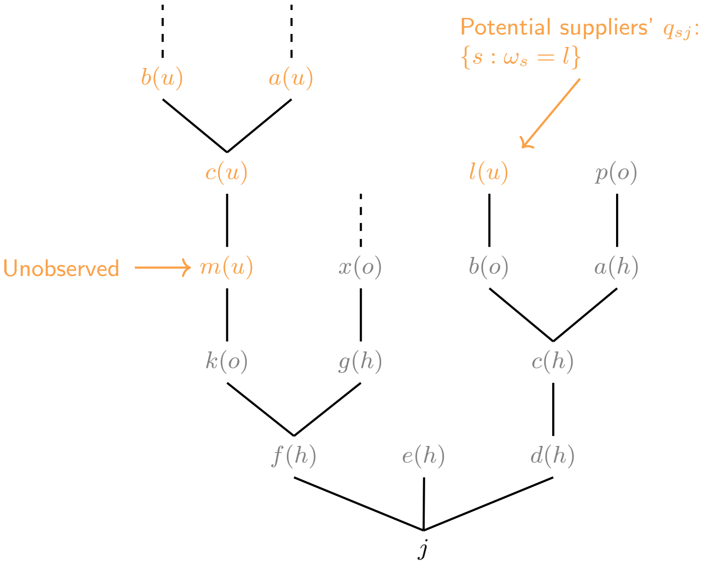
Step 2: unobserved pairs, observed decision - imputed q_{ij}

3. Draw q_{ij} from their respective truncated Pareto distributions for all inputs $\omega \in \mathcal{I}_j \cup \mathcal{O}_j \Rightarrow$ Representative sample.
   - **Outsourced inputs:** For each potential supplier s that was not chosen against supplier i, the conditional distribution is $\frac{1}{q_{sj}} \sim Pareto\left(\frac{c_i(\omega)/q_{ij}}{c_s(\omega)}, \theta\right)$.
   - **In-house inputs:** For each potential supplier s, the conditional distribution is $\frac{1}{q_{sj}} \sim Pareto\left(\frac{c_j^{\mathcal{I}}(\omega)}{c_s(\omega)}, \theta\right)$

$\Rightarrow$ Multiple draws, or directly set $\frac{1}{q_{sj}} \equiv E\left(\frac{1}{q_{sj}} | \frac{c_s(\omega)}{q_{sj}} > \frac{c_i(\omega)}{q_{ij}}\right) = \frac{\theta}{\theta-1}\frac{c_i(\omega)/q_{ij}}{c_s(\omega)}$ and $\frac{1}{q_{sj}} \equiv E\left(\frac{1}{q_{sj}} | \frac{c_s(\omega)}{q_{sj}} > c_j^{\mathcal{I}}(\omega)\right) = \frac{\theta}{\theta-1}\frac{c_j^{\mathcal{I}}(\omega)}{c_s(\omega)}$

Calibration of the q_{ij} I

Step 3: unobserved pairs, unobserved decision - predicted q_{ij}



Calibration of the q_{ij} II

Step 3: unobserved pairs, unobserved decision - predicted q_{ij}

4. For upstream products for which we **don't observe** the fragmentation decision of firm j due to sequentiality:
   PPML [Gourieroux et al., 1984, Silva and Tenreyro, 2006] to estimate the buyer (z_j^b) and the supplier (z_i^s) fixed effects and δ:

$$\ell(\delta, \mathbf{z}^b, \mathbf{z}^s) = \sum_{i=1}^{N_b} \sum_{j=1}^{N_s} \{ z_{ij} \times (z + z_i^s + z_j^b + \mathbf{X}_{ij}\delta) - \exp(z + z_i^s + z_j^b + \mathbf{X}_{ij}\delta) \}$$

   Where $z = log \frac{\theta}{\theta - 1}$ with zero-sum normalization.
   - $\Rightarrow$ Build all the unobserved q_{ij} using the estimated fixed effects and drawing ϵ_{ij} randomly from the $Exp(\theta)$ distribution.

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Alternative calibration method

Selection purely on observables

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1. Assume that selection is fully explained by observables:
   $q_{ij} = exp(z_i^s + z_j^b + \mathbf{X}_{ij}\delta)exp(\varepsilon_{ij})$
   - z_i^s, z_j^b are supplier and buyer FE
   - X_{ij}: distance and degree of input specialization
   - ε_{ij}: iid measurement error *unobserved* by the firm during selection process

$\Rightarrow$ $E(\varepsilon_{ij}|i,j,X_{ij},q_{ij}\text{ observed}) = E(\varepsilon_{ij}|i,j,X_{ij}) = 0$

2. Estimate $\{\{z_i^s\}_i, \{z_j^b\}_j, \delta\}$ using OLS as

$$\begin{aligned} E(log(q_{ij})|i,j,X_{ij},q_{ij}\text{ observed}) &= \hat{z}_i^s + \hat{z}_j^b + X_{ij}\hat{\delta} + E(\varepsilon_{ij}|i,j,X_{ij},q_{ij}\text{ observed}) \\ &= \hat{z}_i^s + \hat{z}_j^b + X_{ij}\hat{\delta} \end{aligned}$$

⚠ Valid if high R^2

3. Build the remaining unobserved q_{sj} as

$$q_{sj} = exp(\hat{z}_s^s + \hat{z}_j^b + \mathbf{X}_{sj}\hat{\delta})$$

4. **Refinement:** allow for non-pecuniary component $\varrho_{ij} \sim Pareto(\varrho_m, \theta)$ independent of q_{ij} (make-or-buy based on $\tilde{q}_{ij} = q_{ij} \times exp(\varrho_{ij})$) and use the truncated Pareto property to match observed decisions (using linkages from cases **??** and **??**)

Alternative calibration method I

Discrete choice model [McFadden, 1974, Berry, 1994, Eaton and Kortum, 2002]

- Use only the q_{ij} of observed transactions and estimate the fixed effects accounting for selection
- Assume $q_{ij} = exp(z_i^s + \varepsilon_{ij})$ for $i \neq j$ and $q_{jj}(\omega) = exp(z + \varepsilon_{jj}(\omega))$ (⚠), where $\varepsilon \sim Gumbel(0,1)$

$$E\left(\max_{i \in \mathcal{S}_\omega} \frac{q_{ij}}{c_i}\right) = E\left(\frac{q_{ij}}{c_i} \middle| \frac{q_{ij}}{c_i} > \frac{q_{sj}}{c_s}, \dots, \frac{q_{ij}}{c_i} > \frac{q_{jj}}{c_j^{\mathcal{I}}}\right)$$

⚠ $\frac{c_j^{\mathcal{I}}(\omega)}{q_{jj}}$ is stochastic and $\frac{c_j^{\mathcal{I}}(\omega) q_{ij}}{q_{jj}}$ is not logit:

$$\frac{c_j^{\mathcal{I}}(\omega)}{q_{jj}} = \left(\int_{\hat{\omega} \in \mathcal{I}_j \cap \Omega_\omega} c_j^{el}(\hat{\omega})^{\frac{-\gamma}{1-\gamma}} d\hat{\omega} + q_{jj}^{\frac{\gamma}{1-\gamma}} \int_{\hat{\omega} \in \mathcal{O}_j \cap \Omega_\omega} \left(\frac{c_i^{\mathcal{O}}(\hat{\omega})}{q_{ij}}\right)^{\frac{-\gamma}{1-\gamma}} d\hat{\omega}\right)^{\frac{1-\gamma}{-\gamma}}$$

- ([Boehm and Oberfield, 2023] don't build $c_s(\omega), c_j^{\mathcal{I}}(\omega)$ but require Cobb-Douglas)

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Alternative calibration method II

Discrete choice model [McFadden, 1974, Berry, 1994, Eaton and Kortum, 2002]

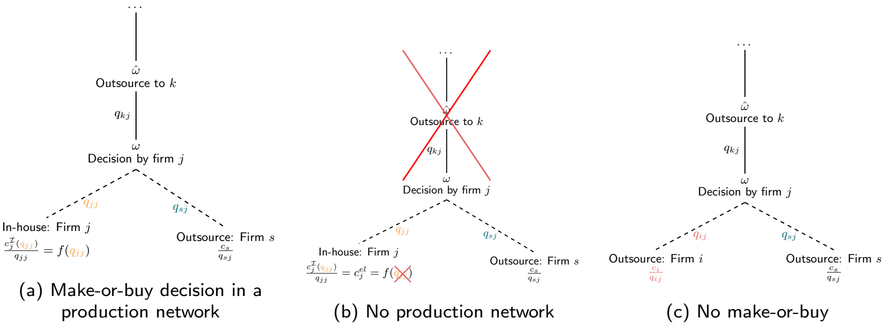

Figure 24: Illustration of the challenges in the calibration of q_{ij}

Exact hat algebra [Dekle et al., 2008] cannot be applied I

Counterfactual aggregate energy intensity:

$$\left(\frac{E}{PY}\right)^{SCC} = \int_N \left(\frac{e_j}{PY}\right)^{SCC} dj$$

Firm-level energy demand:

$$e_j = \frac{1}{p_e} c_j(\omega_j) x_j(\omega_j) \int_{\omega \in \mathcal{I}_j} \frac{p_e e_j(\omega)}{c_j^{el}(\omega) x_j^{el}(\omega)} \frac{c_j^{el}(\omega) x_j^{el}(\omega)}{c_j(\omega_j) x_j(\omega_j)} d\omega$$

Exact hat algebra [Dekle et al., 2008] cannot be applied II

Therefore,

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$$\left(\frac{e_j}{PY}\right)^{SCC} = \frac{1}{p_e^{SCC}} \underbrace{\frac{c_j(\omega_j)x_j(\omega_j)}{PY}}_{\text{Initial cost shares: observed}} \underbrace{\left(\widehat{\frac{c_j(\omega_j)x_j(\omega_j)}{PY}}\right)}_{\text{Intensive + extensive margin}}$$

$$\times \left(\int_{\omega \in \mathcal{I}_j^*} \underbrace{\frac{p_e e_j(\omega)}{c_j^{el}(\omega)x_j^{el}(\omega)}}_{\text{Initial energy share: observed}} \underbrace{\left(\widehat{\frac{p_e e_j(\omega)}{c_j^{el}(\omega)x_j^{el}(\omega)}}\right)}_{\text{Intensive margin: only varies through } p_e} \right.$$

$$\times \underbrace{\frac{c_j^{el}(\omega)x_j^{el}(\omega)}{c_j(\omega_j)x_j(\omega_j)}}_{\text{Initial energy+labor share: observed}} \underbrace{\left(\widehat{\frac{c_j^{el}(\omega)x_j^{el}(\omega)}{c_j(\omega_j)x_j(\omega_j)}}\right)}_{\text{Intensive + extensive margin}}$$

$$\left. + \int_{\omega \in \mathcal{I}_j^{SCC} \& \omega \notin \mathcal{I}_j^*} \underbrace{\left(\frac{p_e e_j(\omega)}{c_j(\omega_j)x_j(\omega_j)}\right)^{SCC}}_{\text{New in-house inputs (zero initial share)}} d\omega \right)$$

Simulating the efficient level of fragmentation (toy model)

1. Simulate fragmentation for $p_e = p_e^{SCC}$ (e.g., 170€ per metric tonne of CO_2 [Pindyck, 2019, EPA, 2022]):

   1.1 *Downstream firms:* For each firm j and for each of its intermediate input $\omega \in \Omega_j$, solve for the cost of producing in-house:

   $$c_j^{\mathcal{I}}(\omega) = \left(\beta_\omega \left(\frac{p_e^{SCC}}{\nu_j} \right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega) w^{\frac{-\rho}{1-\rho}} \right)^{\frac{-(1-\rho)}{\rho}}$$

   1.2 *Upstream firms:* For all the potential suppliers of i (S_i), solve for their unit cost:

   $$c_i^{\mathcal{O}}(\omega) = \left(\beta_\omega \left(\frac{p_e^{SCC}}{\nu_i} \right)^{\frac{-\rho}{1-\rho}} + (1-\beta_\omega) w^{\frac{-\rho}{1-\rho}} \right)^{\frac{-(1-\rho)}{\rho}}$$

   1.3 *Effective unit cost:* $c_j(\omega) = \min \left\{ c_j^{\mathcal{I}}(\omega), \min_{s \in S_i} \frac{c_s(\omega)}{q_{sj}} \right\}$

   1.4 *Fragmentation decision:* $i \in \mathcal{I}_j^{SCC}$ if $c_j(\omega) = c_j^{\mathcal{I}}(\omega)$, $i \in \mathcal{O}_j^{SCC}$ otherwise

   1.5 *Reallocation:* Compare $\mathcal{I}_j^{SCC}$ and $\mathcal{I}_j^*$ for all firms j.

2. Compute $\left(\frac{E}{Y}\right)^{SCC}|_{p_e^{SCC}}, \left(\frac{E}{Y}\right)^{SCC}|_{p_e^*}, \left(\frac{E}{Y}\right)^{*}|_{p_e^{SCC}}$ to compare it to $\left(\frac{E}{Y}\right)^{*}|_{p_e^*}$ (price, persistence, fragmentation effects)

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Outsourcing wedge I

Suppose that firm j has chosen supplier $i = \{b, j\}$. However, the SCC cost comparison yields (b is the most energy-adjusted efficient supplier):

$$\frac{1}{q_j(\omega)\psi_i\tau_{ij}}\left(\beta\Big(\frac{p_e^{SCC}}{\nu_i}\Big)^{\frac{-\rho}{1-\rho}} + (1-\beta)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}$$

$$> \frac{1}{\psi_b\tau_{bj}}\left(\beta\left(\frac{p_e^{SCC}}{\nu_b}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta)w^{\frac{-\rho}{1-\rho}}\right)^{\frac{-(1-\rho)}{\rho}}$$

Where τ_{ij} represents iceberg trade costs (distance), ψ_i is TFP, and $q_j(\omega)$ summarize buyer-input-specific outsourcing factors.

Outsourcing wedge II

$$\text{Wedge:} \quad \mu_{ib,j} = \frac{q_j(\omega)\psi_i\tau_{ij}}{\psi_b\tau_{bj}} \left(\frac{\beta\left(\frac{p_e}{\nu_b}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta)w^{\frac{-\rho}{1-\rho}}}{\beta\left(\frac{p_e}{\nu_i}\right)^{\frac{-\rho}{1-\rho}} + (1-\beta)w^{\frac{-\rho}{1-\rho}}} \right)^{\frac{-(1-\rho)}{\rho}}$$

- $\mu_{ib,j}$ represents the wedge that prevented firm j to choose the most (energy) efficient supplier.
- $\mu_{ib,j}$ represents the relationship-specific subsidy needed to induce the efficient make-or-buy decision and allocation.
- $\mu_{ib,j}$ is a distortion (quasi-rent), hence doesn't enter aggregate outcomes. All outsourcing gains/losses are captured by TFP, trade costs and buyer-input-specific outsourcing factor.
  Counterfactual make-or-buy decision is still based on the μ drawn from the Pareto distributions, as $\mu_{ib,j}$ is an upper bound and only observed for observed decisions.
- Statistics on $\mu_{ib,j}$ by sectors, firm size, upstreamness, etc. inform on the extend of the distortion in the economy without relying on the Pareto assumption Back