The Distributional and Aggregate Effects of Factor-Biased Climate Change

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How does climate change affect us?

- Climate change is a major driver of trends in aggregate economic outcomes
- Climate impacts are also borne unequally across people: across locations, across the income distribution, across sources of income
- We develop a parsimonious framework that allows us to understand why climate impacts are heterogeneous and how that matters for aggregate outcomes

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We ask:

- Does climate shift the labor vs. capital income share?
- Which mechanisms generate heterogeneous factor-share responses?
- How does heterogeneity affect aggregate and distributional projections?

Our Core Finding: Climate Change is Factor-Biased

We establish a new fact:

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1. Climate Damage is Capital-Biased
   - → Climate change damages K-aug. productivity much more than L-aug. productivity
   - → Firms w/ a higher capital share experienced a larger decline in Solow residual
2. Capital-Labor Complementarity
   - → We estimate the micro EoS between K and L via a new granular IV approach
   - → On average, $\hat{\sigma}_{KL} = 0.3$. Strong complementarity in all major sectors.

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Using these estimates, we show that, since 1981, factor-biased climate change

- has reduced aggregate income by 2%; explains 10% (.53 pp) of the global labor share decline.

Contribution to the literature

Many, many climate papers study productivity effects (e.g. Dell, Jones, and Olken 2012; Burke, Hsiang, and Miguel 2015; Somanathan et al. 2021; Nath 2023)

- We show that climate change is factor-biased and that this has significant consequences for the aggregate and distributional effects of climate change

The climate literature has focused on heterogeneity across locations and the income distribution (e.g. Dell, Jones, and Olken 2012; Diffenbaugh and Burke 2019; Carleton et al. 2022), with one paper studying the labor share in the US (Qiu and Yoshida 2024)

- We show there are heterogeneous impacts within a firm and across the world

Recent literature has focused on carefully identifying the K-L elasticity of substitution (e.g. Raval 2019; Oberfield and Raval 2021; Gechert et al. 2022; Berlingieri et al. 2024)

- We provide panel data-based estimates for 32 countries across 9 major sectors, suggesting capital and labor are complements

A deep literature studying the global decline in the labor share (e.g. Karabarbounis and Neiman 2013; Grossman and Oberfield 2022; Hubmer 2023; Karabarbounis 2024)

- We provide evidence that climate change has driven about 1/10th of the fall

Data

- Firm-level microdata from 32 countries: $\approx$ 80 million firm-year obs.
  - $\rightarrow$ Orbis Historic: 1995-2018 for 30 European countries
    - Good coverage of gross output in many countries
    - Quasi-census for Spain and Italy
  - $\rightarrow$ China NBS (1998-2007) + India ASI (1999-2018)
    - Census for "above-scale" manufacturing firms
- Weather and Climate Data: Daily Temperature from ERA5 ($0.1^\circ \times 0.1^\circ$). ▸ map

Measurement

For firm i:

- Capital Expenditure: $rK_{it} = r_{c,s(i)t} K_{it}$

  $\rightarrow$ $r_{c,s(i)t}$: Jorgenson (1963) user cost at the country-sector-year level (KLEMS):

$$r_{c,s(i)t} = \underbrace{p^K_{c,s(i)t}}_{\text{Price of Inv.}} \left[r^c_t + \underbrace{KRP^c_t}_{\text{Risk Premium}\approx 7\%} - (1 - \delta_{c,s(i)t}) \left(\frac{p^K_{c,s(i)t+1}}{p^K_{c,s(i)t}} \right) \right]$$

  $\rightarrow$ K_{it}: deflated book value of fixed assets (Perpetual Inventory Method as robustness)

- Labor Expenditure: wL_{it} as total cost of employees
- Factor Share:

$$s_{Kit} = \frac{rK_{it}}{wL_{it} + rK_{it}} \qquad s_{Lit} = \frac{wL_{it}}{wL_{it} + rK_{it}}$$

For region r:

- We measure regional climate using temperature bins:

$$\mathbf{T}_{r,t} = \left\{ \text{Tbin}^{<-5^\circ C}_{r,t}, \text{Tbin}^{-5\sim 0^\circ C}_{r,t}, ..., \text{Tbin}^{>30^\circ C}_{r,t} \right\}$$

- $\text{Tbin}^b_{r,t}$: number of days in year t in which daily average temperature fell into range b.

Reduced-form Evidence:

Climate Shocks Affects Factor Shares

Extreme heat increases expenditures on K relative to L

- Estimating the effect of temperature on relative factor shares

$$\log\left(\frac{rK_{it}}{wL_{it}}\right) = \sum_{b \in B/(10\sim15^{\circ}C)} \lambda^{b} \times \text{Tbin}_{r,t}^{b} + \delta_i + \eta_{c(r)s(i)t} + \varepsilon_{it}$$

with firm and country-sector-year FE

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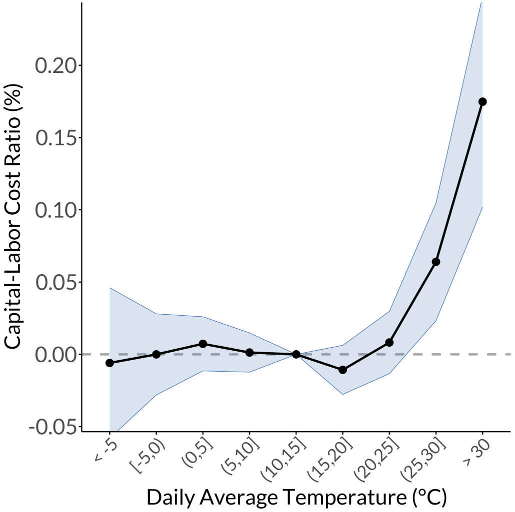
An additional day above 30°C (relative to $10-15^\circ$C) increases the K-to-L expenditure ratio by 0.17%

Labor share of income declines!

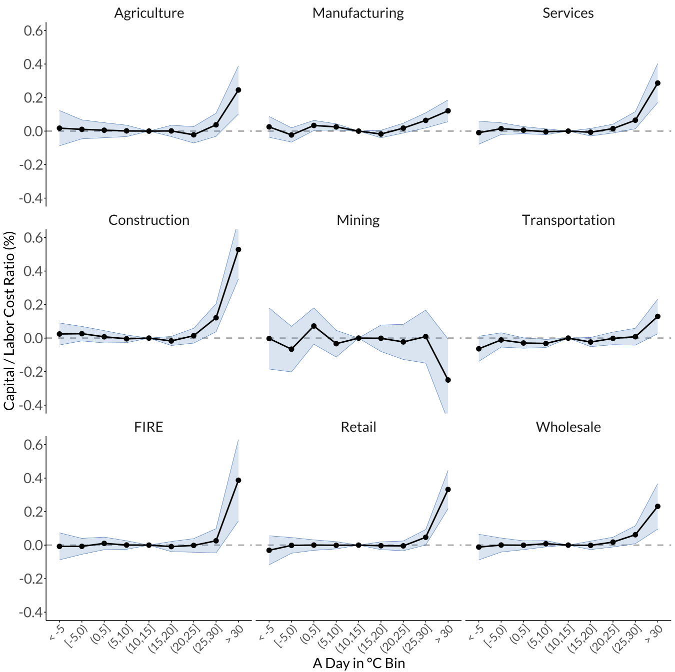
Suggests that historical warming in Southern Europe decreased the labor share by 1 pp



K-L expenditure ratio increases across sectors

Construction, FIRE, Retail, Services: ≈ 0.3 - 0.5% ↑ from a hot day

Ag, Manufacturing, Transportation: ≈ 0.1 - 0.3% ↑



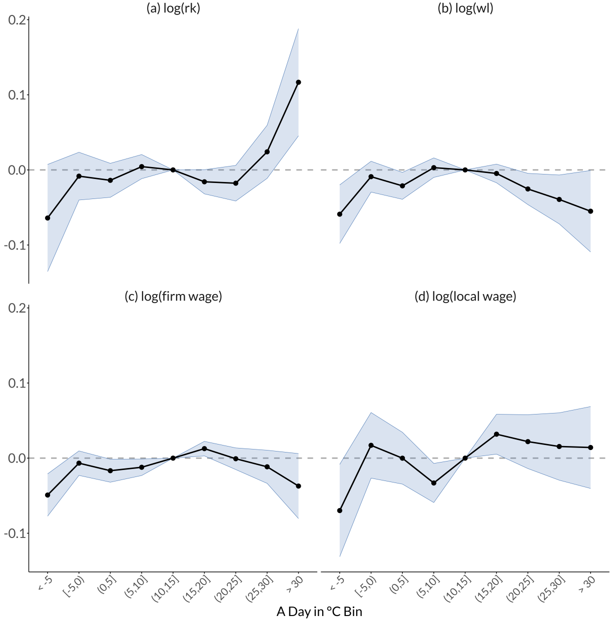
Firms are spending more on capital, less on labor

A shift from labor to capital:

- a large rise in rK (0.12%)
- a smaller drop in wL (-0.05%)

Smaller effects on wages measured at the firm or regional level

Reduced-form results indicate climate change biases factor productivity



The Technical Bias of Climate Damages

Formalizing the notion of factor-biased climate damage

- Suppose a firm with CRS production function $Y_{it} = F_i(A_{it}K_{it}, B_{it}L_{it})$ facing CES demand, and assume K-and-L-augmenting productivity follow

$$A_{it} = \tilde{A}_{it} \exp(\mathbf{a}_{rt} \cdot \mathbf{T}_{rt}), \quad B_{it} = \tilde{B}_{it} \exp(\mathbf{b}_{rt} \cdot \mathbf{T}_{rt})$$

- A firm's Solow residual is

Solow derivation

$$\begin{aligned} d \log \text{Solow}_{it} &= s^K_{it} \cdot a_{it} + s^L_{it} \cdot b_{it} \\ &= \mathbf{b}_{rt} \cdot d\mathbf{T}_{rt} + s^K_{it}(\mathbf{a}_{rt} - \mathbf{b}_{rt}) \cdot d\mathbf{T}_{rt} + s^K_{it} \cdot \tilde{a}_{it} + s^L_{it} \cdot \tilde{b}_{it}, \end{aligned}$$

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Definition

Climate damage is biased toward a factor if heat harms its factor productivity more than others'.

- We define climate damage to be capital-biased if $\frac{d \log(A_{it}/B_{it})}{dT_{rt}^H} < 0$.

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- Climate damage is capital-biased if and only if heat decreases Solow residuals more for firms with higher capital shares:

$$\frac{d \frac{d \log \text{Solow}_{it}}{d\mathbf{T}_{r,t}^H}}{ds_{it}^K} = \mathbf{a}_{rt} - \mathbf{b}_{rt} < 0$$

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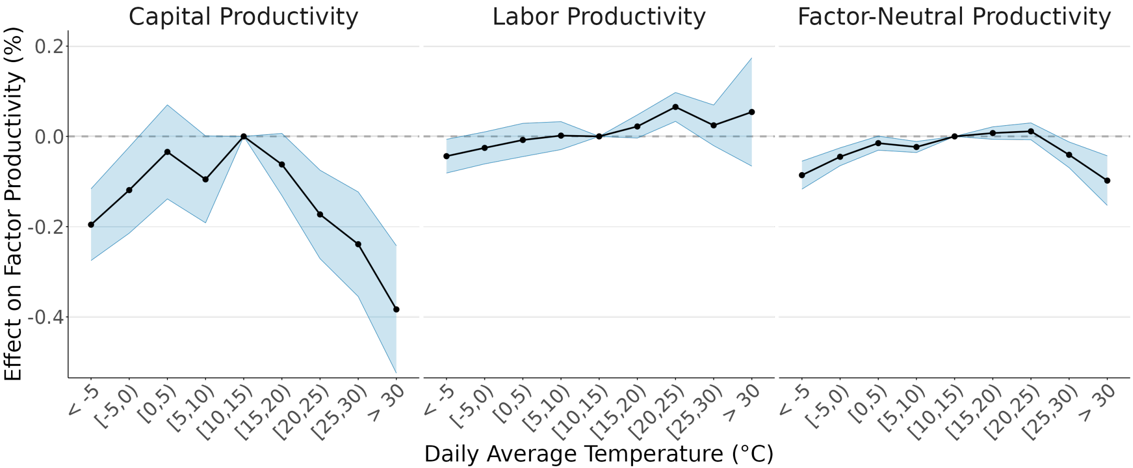
- We can estimate the empirical analogue of the Solow residual equation:

▸ Törnqvist

$$\log\left(\widetilde{\text{Solow}}_{it}\right) = \sum_{b \notin \{10-15^\circ C\}} b^b \cdot \text{Tbin}_{rt}^b + \sum_{b \notin \{10-15^\circ C\}} (a^b - b^b) \cdot s_{it}^K \cdot \text{Tbin}_{rt}^b + \text{controls}_{it} + \text{TWFE}$$

Climate change harms capital, not so much labor



- Heat and cold primarily damage capital, rather than labor.
- Imposing neutrality averages the two, and recovers temperature response functions similar to the existing literature. ▸ Regression Equation

Why does extreme heat damage capital productivity? Some examples...

Derating of Equipment: most equipment is rated for a maximum ambient temperature, must run below nameplate capacity if temperatures are too high

- "Heat Waves Are Shutting Down Data Centers and Breaking the Internet"

  → cooling systems overwhelmed in the UK

- "Heatwave reduces output at Swiss nuclear power plant by 50%"

  → water used for cooling is too hot

Thermal Distress on Structures: Materials such as concrete and steel expand or contract when temperature changes

- "Flights briefly disrupted at UK's Luton airport as heat damages runway"

  → runway buckles

- "Amsterdam cools down its bridges on hot summer days"

  → steel expands

Degradation: high temperatures degrade batteries, worsen the lubrication properties of oils, and deform machinery ac-firms ac-result

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- We introduce the following CES structure with elasticity of substitution σ:

$$Y_{it} = \left[\gamma\left(A_{it}K_{it}\right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)\left(B_{it}L_{it}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

- Firm's optimality conditions imply that:

energy-ces

$$\log(\frac{rK_{it}}{wL_{it}}) = (\sigma-1)\log\frac{w_{r(i)t}}{r_{c,s(i)t}} + (\sigma-1)(\mathbf{a}_r - \mathbf{b}_r)\mathbf{T}_{r,t} + (\sigma-1)\log\frac{\tilde{A}_{it}}{\tilde{B}_{it}} + \log\left(\frac{\gamma}{1-\gamma}\right).$$

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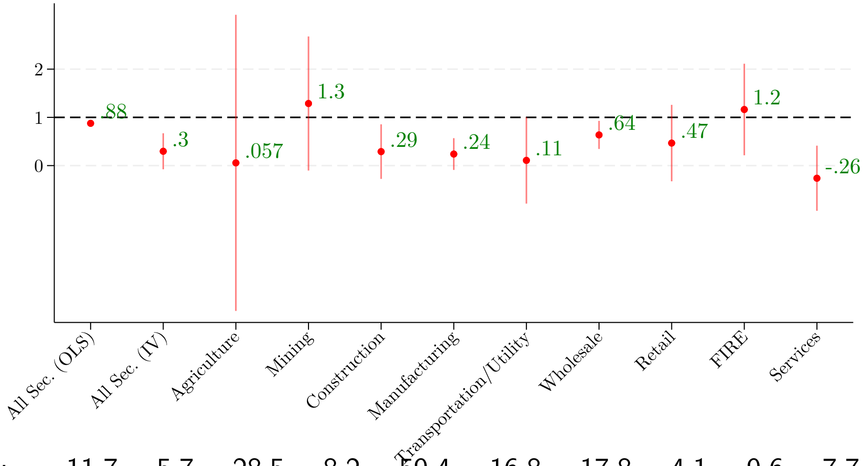
- Construct "Granular Instruments" for wages (Gabaix and Koijen 2024) by summing up idiosyncratic employment shocks of other firms as a regional labor supply shifter: ▸ IV

$$Z^{GIV}_{i,rst} = \sum_{j\in r(i)} \text{Firm Size}_j \times \widehat{\text{Employment Shock}}_{jt}$$

Capital-labor Complementarity is Prevalent in Most Sectors

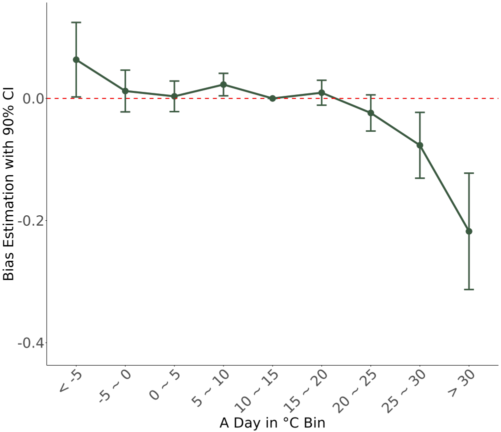


K-P F-stats: 11.7 5.7 28.5 8.2 59.4 16.8 17.8 4.1 9.6 7.7

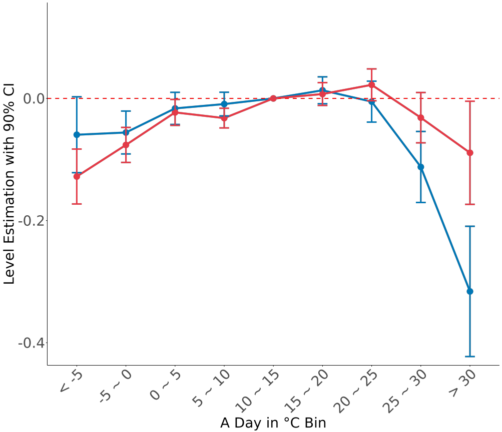
- $\hat{\sigma}_{KL} = 0.3$ for an average firm: K and L are complements (Oberfield & Raval, 2021)
- We can reject Cobb-Douglas for construction, manufacturing, wholesale, and services

Recovering the level effect on capital and labor

(a) Relative Bias Towards Capital:



(b) Level Effects on K- and L- Aug. Prod.



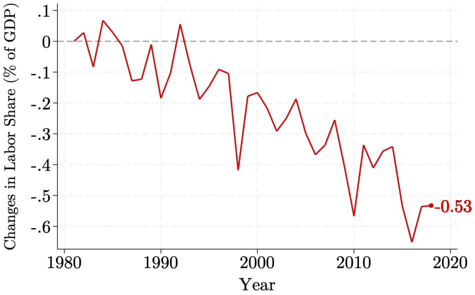
- Extreme heat reduces productivity of both factors, but K-productivity falls much more
- Estimates are broadly similar to the Solow approach

Quantitative Results

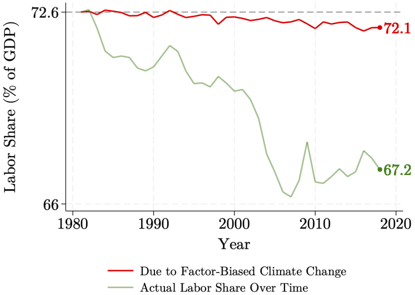
Historical Decomposition of labor share

We apply our ("adaptation"-adjusted) estimates to the historical labor share data:

(a) Climate Impact on Labor Share



(b) Climate Effect vs. Data



- Climate change reduced the labor share by 0.5pp over 40 years
- Explains 10% of the observed 5pp decline (Karabarbounis and Neiman 2013)

Factor bias is critical for quantifying damages

We apply a 2nd-order approximation (Baqaee and Farhi 2019) to Δ region income around the world:

$$\Delta^{\mathbf{T}}_{t,t+1} \log Y_{rt} = \underbrace{\bar{s}^K_{rt}\, \bar{\mathbf{a}}_{rt} \cdot \Delta^{\mathbf{T}}_{t,t+1} \mathbf{T}_{rt} + \bar{s}^L_{rt}\, \bar{\mathbf{b}}_{rt} \cdot \Delta^{\mathbf{T}}_{t,t+1} \mathbf{T}_{rt}}_{\text{1st-order impact}} + \underbrace{\frac{1}{2}\, \bar{s}^K_{rt} \bar{s}^L_{rt} \left(1 - \frac{1}{\sigma}\right) \left[(\bar{\mathbf{a}}_{rt} - \bar{\mathbf{b}}_{rt}) \cdot \Delta^{\mathbf{T}}_{t,t+1} \mathbf{T}_{rt} \right]^2}_{\substack{\text{2nd-order impact} \\ \text{complementarity amplifies uneven factor damage}}}$$

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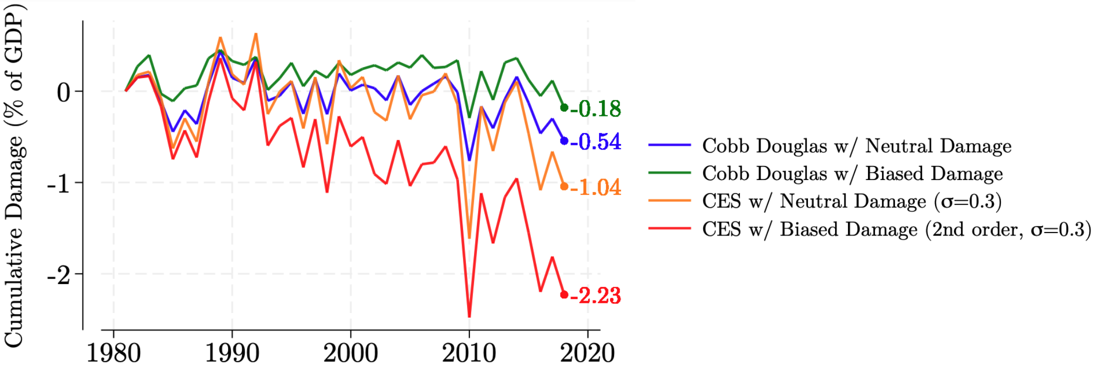
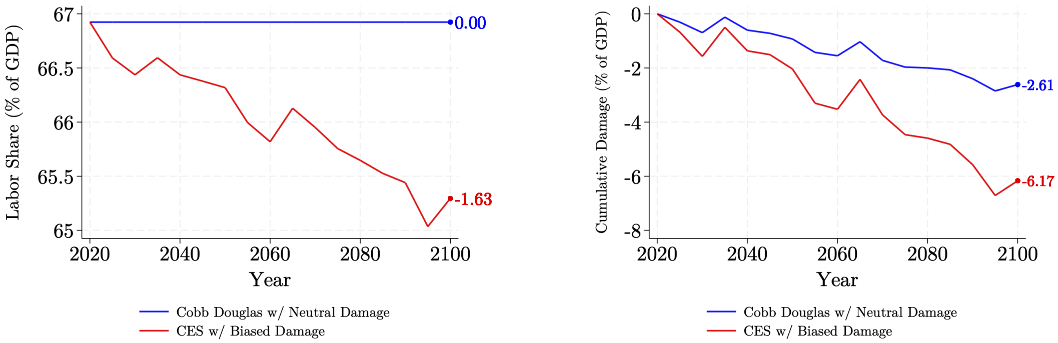
complementarity amplifies uneven factor damage



- Second-order effects due to factor bias more than double estimated impacts
- Conventional approaches underestimate the aggregate effect by 75%.

Projected Change in Labor Share and Aggregate Income in 2100 w/ RCP 4.5

(a) Labor Share (b) Aggregate Damage



Our estimates project a 1.6pp drop in the global labor share and a 6.2% loss in global aggregate income by 2100

Conclusion

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  - → K-aug. Productivity damage is the only way to get a factor-income effect in a large class of growth models (Uzawa), including all IAMs
  - → L-aug. productivity damage → temporary setback along the original BGP → no factor share effect!
  - → Assuming climate neutrality eliminates second-order effects that may be substantial

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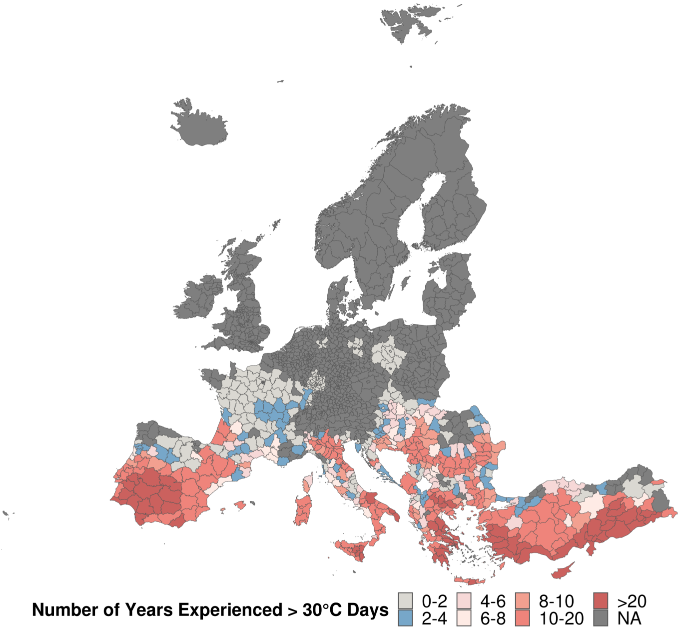
  - → Assuming climate neutrality eliminates second-order effects that may be substantial
- Next Step: from functional distribution of income to actual distribution of income
  - → How does the factor bias interact with heterogeneity in both labor supply and capital portfolio holdings across households?
  - → Income inequality in the long run under climate change?

Appendix

Regions Ever Experienced $\geq$ 30°C days from 1998 to 2018 Back



Empirical Strategy Back

- Estimating the effect of temperature on relative factor shares:

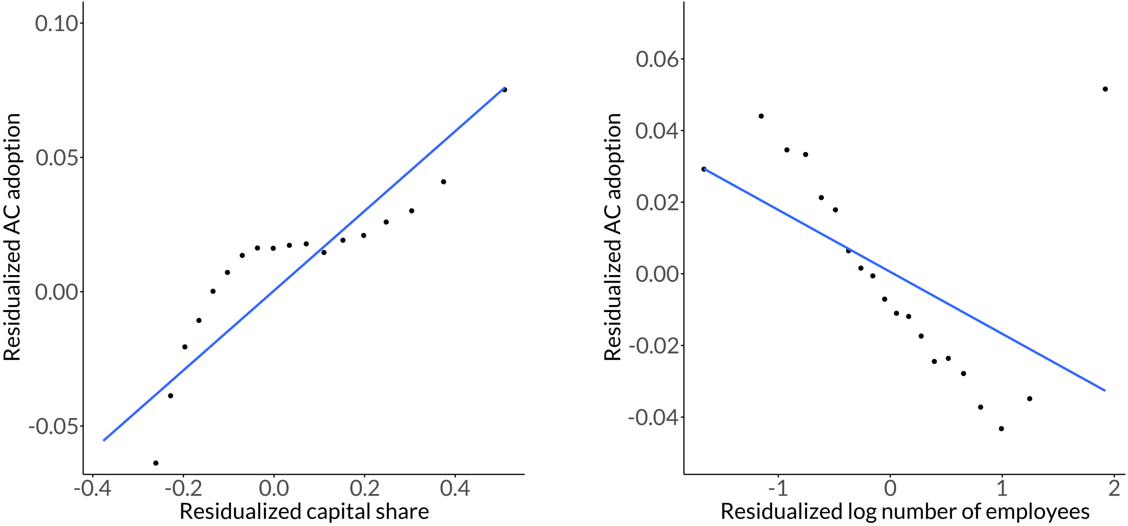
$$\log\left(\frac{rK_{it}}{wL_{it}}\right) = \sum_{b\in B/(10\sim 15^\circ C)} \lambda^b \times \text{Tbin}^b_{r,t} + \delta_i + \eta_{c(r)s(i)t} + \varepsilon_{it}$$

- Firm + country-sector-year FEs
- Standard errors clustered at the regional level.
- λ^b: the average effect of replacing a day in the $10 \sim 15^\circ C$ range with a day in temperature range b on the annual (log) capital-to-labor expenditure ratio

AC Adoption and Firm Characteristics

Back

- Capital-intensive firms and firms with fewer employees are more likely to adopt AC.

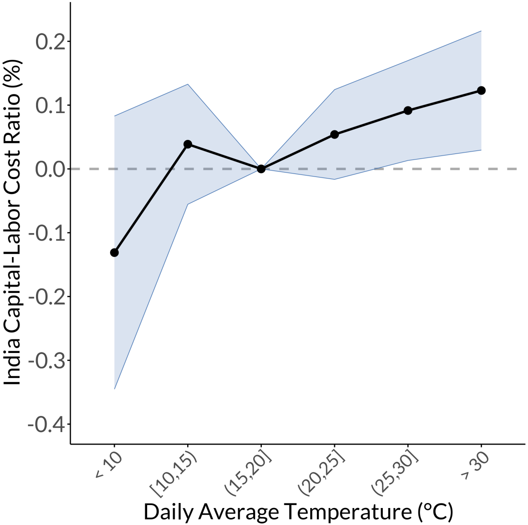


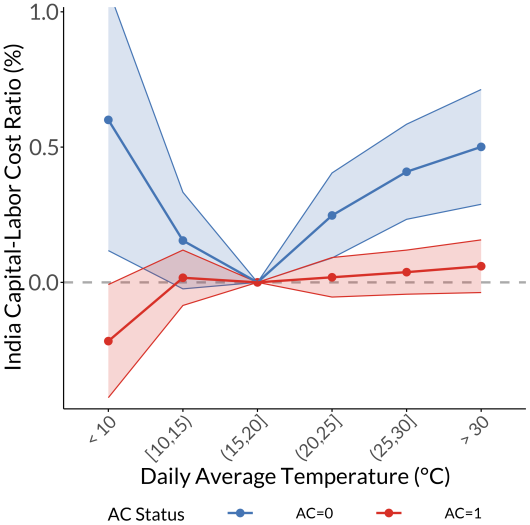
Notes: Conditional on 3-digit industry fixed effects and variables are residualized by firm sales.

Air-Conditioning has no impact on K-L cost ratio for Indian firms Back

- Is the increase in capital expenditure mainly on air-conditioning? Not really.

Figure: Estimates of Air Conditioning's Effect on Temperature Shocks





Solow Derivation Back

- A large set of I^c firms operating in a country c.
- Each firm i resides in a specific region r of c and produces a differentiated output Y_{it}:

$$Y_{it} = F_i(A_{it}K_{it}, B_{it}L_{it})$$

- F_i can be firm-specific CRTS.
- $A_{it} = A_i(\mathbf{T}_{rt}, \cdot)$ and $B_{it} = B_i(\mathbf{T}_{rt}, \cdot)$ are K- and L-augmenting productivity.
- Dixit-Stiglitz demand in a country c : $Y_t^c = \left(\sum_{i \in I^c} Y_{it}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$.
- Envelope theorem implies that:

$$d\log Y_{it} = s_{it}^K\,(d\log A_{it} + d\log K_{it}) + s_{it}^L\,(d\log B_{it} + d\log L_{it})$$

$$d\log \text{Solow}_{it} = s_{it}^{K,cost}\,d\log A_{it} + s_{it}^{L,cost}\,d\log B_{it}$$

where $d\log \text{Solow}_{it}$ is the change in Solow (1957) residuals.

Measuring Solow Residuals via Törnqvist-Index-Based Productivity Back

- The revenue-based index calculates output growth minus the share-weighted input growth.
- **Törnqvist (Divisia approximation) firm input-growth index:**

$$\Delta \ln Q_{it} = \bar{s}_{K,it}(\log K_{it} - \log K_{i,t-1}) + \bar{s}_{L,it}(\log L_{it} - \log L_{i,t-1}),$$

$$\bar{s}_{K,it} = \tfrac{1}{2}(s_{K,it} + s_{K,i,t-1}), \qquad \bar{s}_{L,it} = \tfrac{1}{2}(s_{L,it} + s_{L,i,t-1}).$$

- **Productivity growth (Solow residual growth):**

$$\Delta z_{it} \equiv (\log Y_{it} - \log Y_{i,t-1}) - \Delta \ln Q_{it}.$$

- **Normalization & forward propagation:** set $z_{i,t_0(i)} = 1$ at first non-missing year $t_0(i)$; for $t > t_0(i)$,

$$z_{it} = z_{i,t-1} + \Delta z_{it} \quad \text{(normalization is innocuous with firm FE).}$$

Mapping to Solow Estimation

Back

Firm i's Solow residual can be written as

$$d \log \text{Solow}_{it} = \mathbf{b}_{rt} \cdot d\mathbf{T}_{rt} + s_{it}^{K}(\mathbf{a}_{rt} - \mathbf{b}_{rt}) \cdot d\mathbf{T}_{rt} + \; d\varepsilon_{it} + d\theta_{ct}$$

We estimate its empirical counterpart:

$$\log\left(\widetilde{\text{Solow}}_{cit}\right) = \sum_{b \notin \{10 \sim 15^\circ C\}} b^b \cdot \text{Tbin}_{rt}^b + \sum_{b \notin \{10 \sim 15^\circ C\}} (a^b - b^b) \cdot s_{it}^K \cdot \text{Tbin}_{rt}^b$$
$$+ (a^{10\sim 15^\circ C} - b^{10\sim 15^\circ C}) \cdot s_{K_{it}} + \delta_i + \theta_{cst} + \varepsilon_{it},$$

Empirical Estimation of EoS and Bias

Go Back

We now jointly estimate σ and $\mathbf{a}_r - \mathbf{b}_r$ under CES production using the empirical analogue:

$$\log\left(\frac{r_{it}K_{it}}{w_{it}L_{it}}\right) = \beta \log w_{rt} + \sum_b \beta^b T^b_{r,t} + \theta_{cou,s,t} + \gamma_i + \varepsilon_{it}$$

- $\beta = \sigma - 1$ and $\beta^b = (\sigma - 1)(a^b_r - b^b_r)$
- Identification of β^b follows from the conditional exogeneity of weather fluctuations
- Identification of β requires plausibly exogenous variation in regional wages:

→ Construct "Granular Instruments", $Z^{GIV}_{i,rst}$, for wages: summing up idiosyncratic employment shocks of other firms as a labor supply shifter

$$\log(L_{it}) = \iota_i + \theta_{r,s,t} + \underbrace{u_{it}}_{\text{Granular Shocks}}, \quad Z^{GIV}_{i,rst} = \underbrace{\sum_{j \in I(r,t), j \neq I(r,s(i),t)} s^r_{jt}\hat{u}_{jt} - \frac{1}{|I(r,t)|}\sum_{j \in I(r,t)} \hat{u}_{jt}}_{\text{Leave-one-out weighted sum relative to unweighted average}}$$

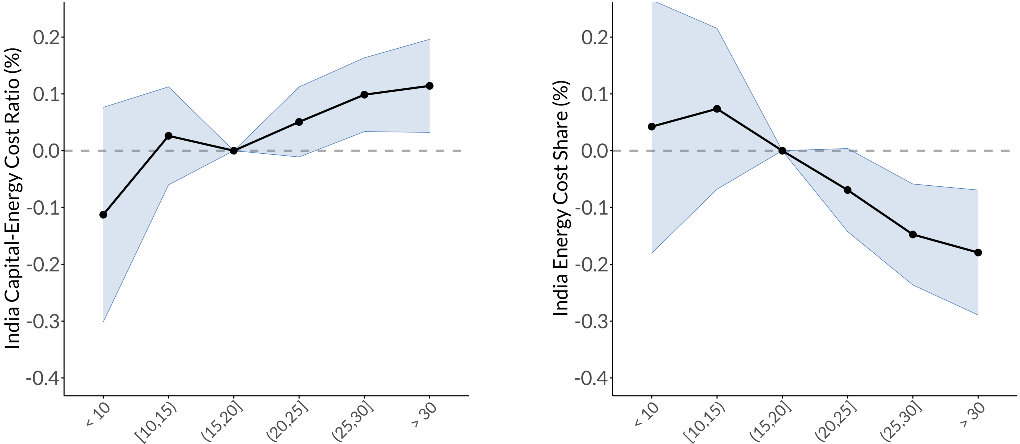
→ Identification Assumption: The expansion of a local Costco (relative to Walmart and Target) is independent to the productivity shock of McD ($\text{Cov}(\varepsilon_{it}, u_{jt}) = 0$).

CES Production with Energy Inputs Back

- In a nested CES: capital–energy bundle combined with labor in an outer CES

$$Y_{it} = \left[\phi Z_{it}^{\frac{\sigma-1}{\sigma}} + (1-\phi)\left(B_{it}L_{it}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \quad Z_{it} = \left(\alpha\left(A_{it}K_{it}\right)^{\frac{\eta-1}{\eta}} + (1-\alpha)E_{it}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

- Capital-energy cost ratio and energy cost share decreases under negative A_{it} damage.



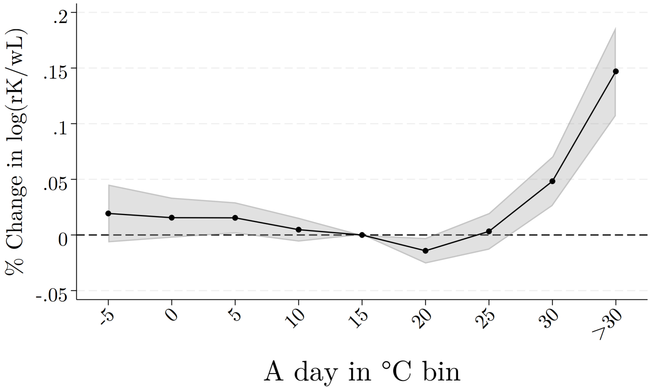
Appendix: Perpetual Inventory Method Robustness

Robustness: PIM-based capital stock

Outcome: capital–labor cost ratio

We re-estimate the baseline reduced-form using PIM-based capital expenditure:

$$\log\left(\frac{rK_{it}^{\text{PIM}}}{wL_{it}}\right) = \sum_{b\in B/(10\sim15^\circ C)} \lambda^b \,\text{Tbin}_{r,t}^b + \delta_i + \eta_{c(r)s(i)t} + \varepsilon_{it}.$$



Robustness: PIM-based capital stock

Outcome: elasticity of substitution (capital vs. labor)

We re-estimate the production-side elasticity using PIM-based capital expenditures, comparing OLS and IV specifications.

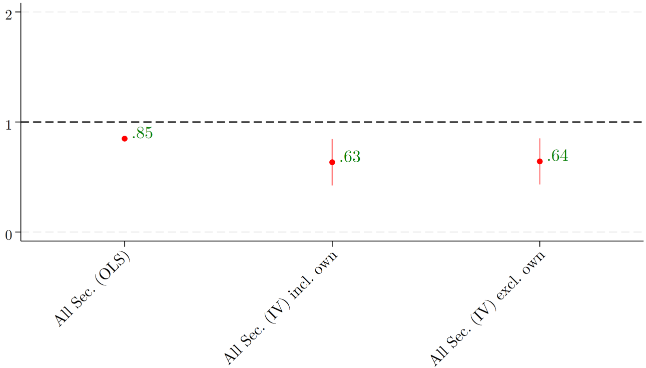


Figure: Estimates for elasticity of substitution using PIM capital stock.

Takeaway: Estimated capital–labor substitutability is close to the baseline.

Appendix: Solow Robustness Results

Solow robustness: average capital share

Concern: time-varying capital shares may respond to temperature shocks.

Check: replace time-varying s_{it}^K with firm-specific average share $\overline{s_i^K}$ to interpret heterogeneity by typical capital intensity.

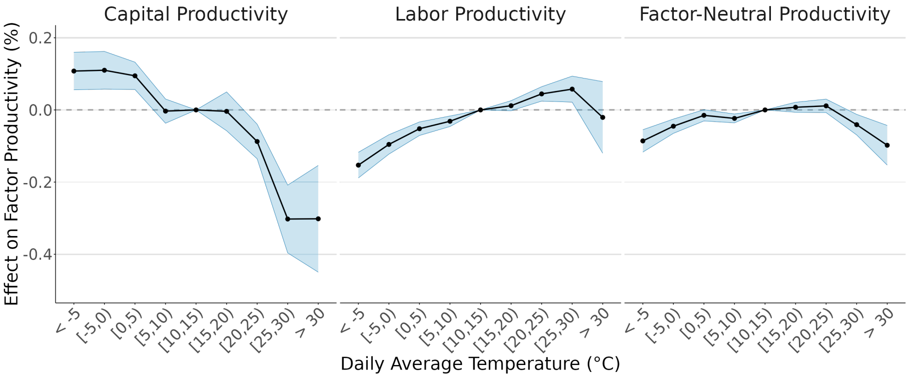


Figure: Solow residual estimation with average capital share.

Takeaway: Extreme heat implies larger losses for capital-augmenting productivity than for labor-augmenting productivity, as in the baseline.

Solow robustness: lagged capital intensity

Concern: contemporaneous capital intensity may mechanically move with shocks.

Check: interact temperature bins with predetermined capital intensity (lagged capital, $L.k_{it}$).

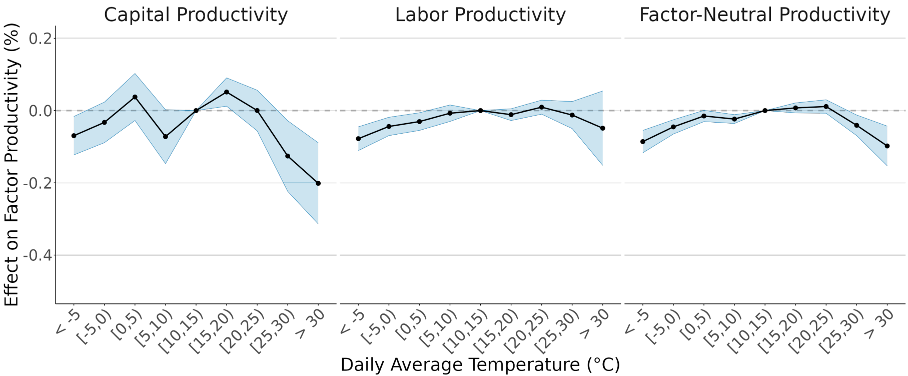
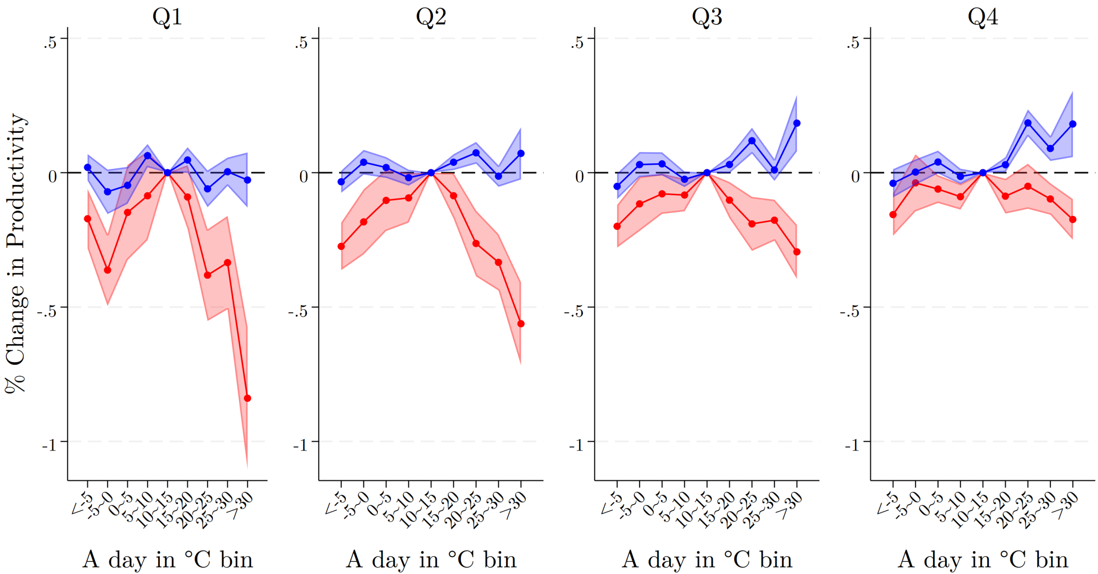


Figure: Solow residual estimation with lagged capital intensity.

Takeaway: Losses under extreme heat are larger for more capital-intensive firms.

Solow robustness: heterogeneity by firm size

Check: split firms into quartiles by sales within country–sector–year.



Solow robustness: sample composition (leave-one-country-out)

Check: re-estimate the Solow specification 32 times, each time excluding one country, and overlay all estimates.