Modeling Firms and Firm Growth

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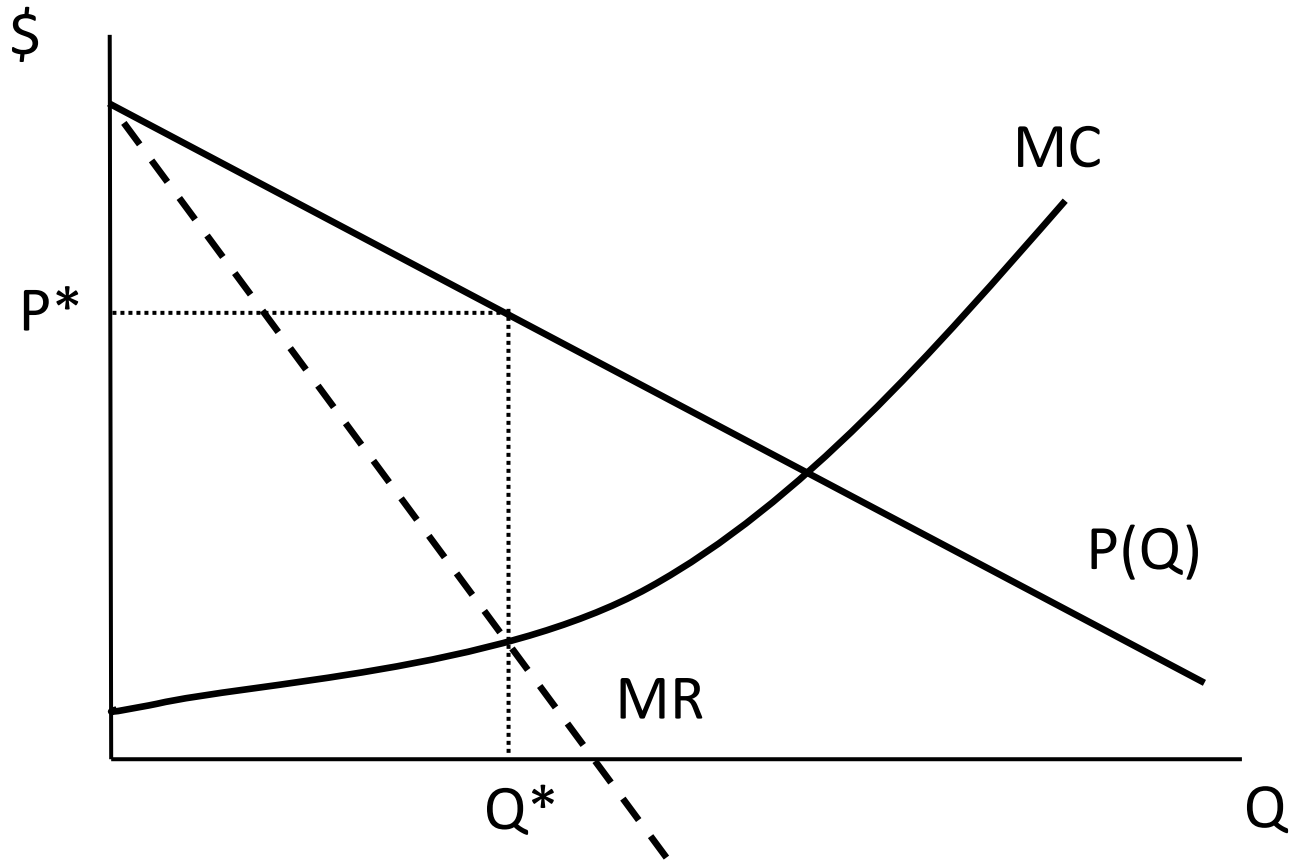
Firms in Equilibrium

Firms' equilibrium outcomes (size, scope, prices, employment, etc.) generally depend on primitives and nature of assumed interactions with other firms

- Primitives
  - Demand
  - Technology
  - Factor prices

- Interactions
  - E.g., Monopoly, monopolistic competition, Bertrand oligopoly, etc.

Firms in Equilibrium: Almost Everything Is a Version of This



Firms in Equilibrium: Some Things to Remember

- Something has to pin down size: downward-sloping demand and/or upward-sloping marginal costs
- *Residual* demand, not market demand, matters for firms (assuming no collusion)
- Need some sort of entry condition to pin down industry equilibrium
  - Equilibria are tricky to compute if they depend on aggregates of choices
- (Lots of) Heterogeneity in primitives is the rule, not the exception
  - Heterogeneity can come from technology/productivity, demand, or factor prices
  - Changes in primitives are responsible for a lot of growth, but shifts in aggregates or competitors' primitives can matter too
- Making primitives endogenous is hard when there are interactions among firms (i.e., anything except single-agent optimizations)
  - Optimal forward-looking actions depend on beliefs about other firms' actions

Heterogeneity

Ubiquitous: Huge size differences, even within narrowly defined industries

Example: U.S. ready-mixed concrete (NAICS 327320) in 2022; about 1950 firms

Employment range	Number of Firms	Plants per Firm	Revenues per Firm ($ million)	Revenues per Plant ($ million)
<5	470	1.0	1.2	1.2
5-9	290	1.0	2.6	2.6
10-19	400	1.1	5.5	5.0
20-99	560	2.0	17.0	8.6
100-499	160	6.3	58.5	9.3
500+	60	42.5	319.9	7.5
All	1950	3.1	21.8	7.1

Heterogeneity

Example: U.S. ready-mixed concrete (NAICS 327320) in 2022; 2000 firms

X Largest Firms	Avg Plants per Firm	Avg Revenues per Firm ($ million)	Avg Revenues per Plant ($ million)
All	3.1	21.8	7.1
50	54	431.6	7.9
20	102	755.9	7.4
8	137	1182	8.6
4	187	1611	8.6

Heterogeneity

Where does this enormous size heterogeneity come from?

Huge productivity (TFP) differences:

- Typical plant-level 90-10 percentile total factor productivity ratio within 6-digit NAICS U.S. manufacturing industries is 2-to-1 (ready-mixed concrete is 1.95)
- Evidence indicates even larger in developing countries
  - E.g., 90-10 TFP within 4-digit mfg industries in LAC = 6.7! (Garone et al., 2020)
- Huge gaps outside manufacturing too
  - E.g., Medicare enrollees treated for a heart attack at 90th percentile hospital within U.S. live 55% longer than similar patient at 10th percentile hospital

Heterogeneity

An enormous literature has documented that these productivity variations are related to other producer-level outcomes

- Survival
- Faster future growth
- Higher wages
- Lower quality-adjusted prices

Results found in an enormous assortment of industries, countries, and time periods

Heterogeneity

But it not just productivity (in fact it's maybe not even mostly productivity)

Even within commodity-like product industries (like ready-mixed), moving 1 SD in the TFP distribution affects 5-year survival probability only *one-fourth* as much as moving 1 SD in "idiosyncratic demand" distribution

- Demand = difference in log quantity sold if everyone in industry charged same price

Atkin, Khandelwal, and Osman (2017): Demand subsidy was the impetus to export and growth in ways not possible with supply subsidy

Discussion of demand-based constraint in Woodruff (2018)

Heterogeneity

The supply (TFP) versus demand distinction raises measurement problems in most micro datasets

Typically the only measure of firm-level output is revenues, but productivity is Q/Inputs; generally, revenues are not the right measure of Q

- Price variation across firms that does not reflect units of quality moves around productivity measures but is not correlated with firm's ability to turn inputs into utility-delivering products

Solutions: work with data where P and Q are separately available (and Q is appropriate output measure), or impose demand system and jointly estimate PF and demand

A Model of Heterogeneous Producer Industry: Demand

Industry comprises a continuum of producers of measure N; monopolistic competition

Each makes a differentiated variety of the industry good (indexed by i)

Utility: $U = y + \alpha \int_{i \in I} q_i di - \frac{1}{2}\left(\eta + \frac{\gamma}{N}\right)\left(\int_{i \in I} q_i di\right)^2 + \int_{i \in I} \delta_i q_i di - \frac{1}{2}\gamma\left(\int_{i \in I}(q_i - \bar{q})^2 di\right)$

where $\bar{q} \equiv \frac{1}{N}\int_{i \in I} q_i di$, $\alpha > 0$, $\eta > 0$, and $\gamma \geq 0$; δ_i is firm *i*'s idiosyncratic demand

- Notice: γ forces curvature across varieties
- Higher γ → consumers less responsive to price differences ("lower substitutability")
- $\gamma = 0$ means goods are prefect substitutes

A Model of Heterogeneous Producer Industry: Supply

Production uses a single input x available at exogenous firm-specific price w_i; firms have productivity level ω_i

Production function: $q_i = \omega_i x_i$

Firm's total costs are therefore $\mathrm{C}(q_i) = \left(\frac{w_i}{\omega_i}\right) q_i$

A Model of Heterogeneous Producer Industry: Profits

Given demand curve and cost structure, a firm's profit-max price is

$$p_i = \frac{1}{2}\frac{\gamma}{\eta N + \gamma}\alpha - \frac{1}{2}\frac{\eta N}{\eta N + \gamma}\left(\bar{\delta} - \bar{p}\right) + \frac{1}{2}\left(\delta_i + \frac{w_i}{\omega_i}\right)$$

Its maximized profits are: $\pi_i = \frac{1}{4\gamma}\left(\Theta + \delta_i - \frac{w_i}{\omega_i}\right)^2$

Θ is a collection of industry-wide parameters

Define the "profitability index" $\phi_i \equiv \delta_i - \frac{w_i}{\omega_i}$

A Model of Heterogeneous Producer Industry: Selection

Imposing that no firm operates at a loss, there will be a value of ϕ_i where profits are zero

Label this "cutoff profitability index" ϕ^*

Easy to show through substitution that any firm's operating profits are

$$\pi_i = \frac{1}{4\gamma}(\phi_i - \phi^*)^2$$

A Model of Heterogeneous Producer Industry: Entry

Large pool of ex-ante identical potential entrants decides whether to pay sunk entry cost s to take primitives draw from a distribution

- In principle this is a joint distribution of δ_i, ω_i, and w_i, but for simplicity suppose it is a single-dimensional profitability distribution $G(\phi)$, $\phi \in [\phi^L, \infty)$

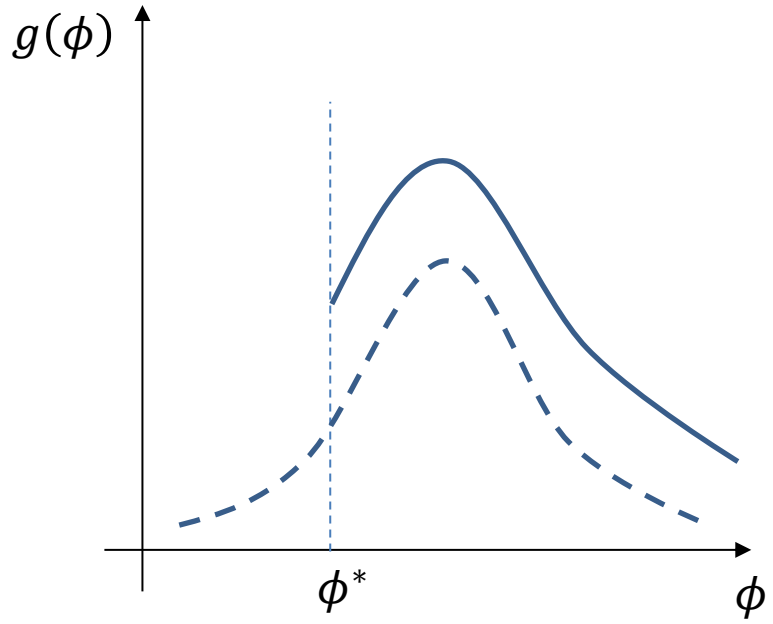
Free entry condition: Potential entrants who pay s receive ϕ draw and then determine whether to begin production and earn operating profits as above; because only potential entrants with draws $\phi > \phi^*$ will choose to produce in equilibrium, the expected value of entry (equal to zero) is

$$V^e = \int_{\phi^*}^{\infty} \frac{1}{4\gamma} (\phi_i - \phi^*)^2 \, g(\phi) d\phi - s = 0$$

A Model of Heterogeneous Producer Industry: Eqm. Selection

Free entry condition embodies the two boundary conditions of the model: no producer operates at a loss, and entry occurs until expected value is zero

Endogenous selection: equil profitability distribution is truncated version of $G(\phi)$:



A Model of Heterogeneous Producer Industry: Comparative Statics

The free entry condition summarizes the two boundary conditions of the model: no producer operates at a loss, and entry occurs to the point where expected value is zero

Comparative statics:

$$\frac{d\phi^*}{d\gamma} = \frac{-\partial V^e / \partial \gamma}{\partial V^e / \partial \phi^*} = \frac{\overbrace{\int_{\phi^*}^{\infty} \frac{1}{4\gamma^2} (\phi_i - \phi^*)^2 \, g(\phi) d\phi}^{>0}}{\underbrace{-\int_{\phi^*}^{\infty} \frac{1}{2\gamma} (\phi_i - \phi^*) \, g(\phi) d\phi}_{<0}} < 0$$

I.e., higher substitutability (lower γ) raises the cutoff profitability level ϕ^*

A Model of Heterogeneous Producer Industry: Comparative Statics

Also:

$$\frac{d\phi^*}{ds} = \frac{-\partial V^e/\partial s}{\partial V^e/\partial \phi^*} = \frac{1}{\underbrace{-\int_{\phi^*}^{\infty} \frac{1}{2\gamma}(\phi_i - \phi^*)\, g(\phi) d\phi}_{<0}} < 0$$

I.e., higher entry costs reduce the cutoff profitability level ϕ^*

BTW if there is a fixed *operating* cost, the comparative static has the opposite sign: higher fixed operating costs raise ϕ^*

- Opposite sign from entry cost because of what's known when decisions are made

A Model of Heterogeneous Producer Industry: Firm Size

What about size? Equilibrium output is:

$$q_i = \frac{1}{\gamma}(\Theta + \phi_i)$$

I.e., firms with higher demand, higher productivity, or lower factor prices are larger

But Θ decreases in competitors' avg demand and productivity (and increasing in avg factor prices), so for a given ϕ_i having stronger competitors makes a firm smaller

- Higher α (avg demand level for industry) makes all firms larger

BTW, firm's price increases in demand and factor prices and decreases in productivity

A Model of Heterogeneous Producer Industry: Allocation

Equilibrium output is $q_i = \frac{1}{\gamma}(\Theta + \phi_i)$

Important: $\frac{dq_i}{d\phi_i d\gamma} < 0$

I.e., more substitutable outputs make firm size more sensitive to the profitability draw

I.e., **allocation** is more responsive to type

- Firm size distribution is transformation of type distribution that depends on parameters

A Model of Heterogeneous Producer Industry: Reallocation

Static model, but imagining dynamics as moving from one steady state to the next implies firm growth comes from:

- Changes in its primitives
- Changes in competitors' primitives
- Industry-level shocks

Same implications in more explicitly dynamic models (e.g., Ericson and Pakes, 1995, Asplund and Nocke, 2006)

The first two are about relative share changes in the model—**reallocations**

A Model of Heterogeneous Producer Industry: Misallocation

Note that the size heterogeneity (and if you pick parameters, the model can have a lot of heterogeneity) need not lead distribution to be inefficient

- Firms can have (much) lower productivity than others, but under some conditions they enter and are the right size—factors' marginal revenue products are equal

But suppose some of the allocation is wrong: some firms are too big or too small for their type; MRPs are not equal. This is **misallocation**

- Dhingra and Morrow (2019) and Bagwell and Lee (2023) talk about this

In misallocation models (e.g. Hsieh and Klenow, 2009), "wedges"/frictions are primitives that affect entry and equilibrium size distribution but prevent equal MRPs

Where Does the Heterogeneity Come From?

This framework raises obvious questions

- Where do the differences in primitives come from in practice?
- Are they exogenous or endogenous?
- What mechanisms?
- Which are misallocations versus not?

Where Does the Heterogeneity Come From?

On productivity, from Syverson (2011):

Two broad sets of factors affect equilibrium productivity distribution

1. Things that, at least in concept, are within a businesses' control—"levers"
    - These mostly affect individual ϕ_i
2. Aspects of the operating environment—"external factors"
    - These mostly affect selection margin but sometimes also individual ϕ_i

Where Does the Heterogeneity Come From?

Levers

1. Managerial practices/talent
2. Higher-quality labor and capital
3. IT and R&D
4. Learning-by-doing
5. Product innovation
6. Firm structure decisions

Where Does the Heterogeneity Come From?

External factors

1. Productivity spillovers
2. Competition—both intra-market and through international trade
3. Regulatory environment
4. Input market flexibility

Size and Productivity, or Size and “Productivity”?

Some caution (or subtlety) is warranted in thinking about the evidence linking firm size and growth to productivity

Remember discussion above how measured productivity often uses revenues/input

Revenue variation can come from P variation not tied to output/input

This can be one role for demand primitive, but can also reflect the intensity of competition

Demand

Several of the levers can apply to demand as well as productivity:

1. Managerial practices/talent ✓
2. Higher-quality labor and capital ~ ✗
3. IT and R&D ~
4. Learning-by-doing ✗
5. Product innovation ✓ ✓
6. Firm structure decisions ✓

Demand

But there are also factors tied closely to demand not in productivity list

- Installed customer base
- Brand
- Reputation
- Product variety (though scope economies are productivity)

Demand

As mentioned, these demand factors matter a lot in predicting size and survival, even in commodity-like product industries

Also matters for growth—Foster, Haltiwanger, and Syverson (2016):

TABLE 1
EVOLUTION OF PRODUCTIVITY AND DEMAND ACROSS PLANT AGES

	Plant age dummies			
Variable	Entrant	Young	Medium	Exiter
Productivity (TFPQ)	0.021 (0.008)	0.010 (0.009)	−0.002 (0.009)	−0.020 (0.008)
Demand	−0.892 (0.040)	−0.481 (0.043)	−0.381 (0.046)	−0.590 (0.038)

Demand

In Foster, Haltiwanger, and Syverson (2016), we ran a horse race between exogenous and endogenous demand accumulation

Exogenous: Demand accrues over time regardless of actions of firm

Endogenous: Firm must take action for demand to accrue; more action, more demand accrual (we model action as sales; more sales today → demand shifts out tomorrow)

Results indicate almost all accrual is endogenous

Competition

As noted, competition can influence equilibrium type (and therefore size distribution)

Two mechanisms of action

- Existing businesses spurred to improve type

  Hicks's "best of all monopoly profits is a quiet life"

- Selection/Darwinian survival: low type shrink or go out of business, high type enter and grow

Both matter, but relative importance varies across settings

- E.g., Backus (2020): ready-mixed is mostly induced improvement

Competition

In our monopolistically competitive model, competition is summarized in substitutability, γ

More substitutability → flatter residual demand curves and output is more responsive to cost differences from ω_i or w_i (γ = 0 is perfect competition); δ_i shifts out residual demand curve

- Residual demand curve slope is useful for thinking about competiveness of markets

Competition

But more explicitly modeled competitive structures (e.g., Cournot, differentiated-product Bertrand) also have same implication that stronger competition flattens residual demand curves

Removal of frictions can also increase competition and have same effects

- Search costs
- Switching costs
- Transport/travel costs
- Trade costs

Conclusion

We could keep going, but you'll see more detail on a lot of these issues throughout this course

Firm size in equilibrium depends on heterogeneous primitives and firm interactions

Heterogeneity has many sources and goes into both supply and demand sides of firm's environment

An important component of market interaction, and that has notable influence on firm interactions and, sometimes, primitives too